ON THE APPLICABILITY OF MacADAM ELLIPSES TO SMALL COLOR DIFFERENCES IN WOOL

G. J. PATTON

ON THE APPLICABILITY OF MacADAM ELLIPSES TO SMALL COLOR DIFFERENCES IN WOOL

A Thesis presented to the Faculty of the Lowell Textile Institute as a partial requirement for the degree of Master of Science in Textile Engineering

TABLE OF CONTENTS

ACKNOWLEDGMENT

An expression of sincere appreciation by the author is tendered to Professor Harry C. Brown for his direction, criticism, and advice and to Mr. Roland E. Derby, Jr. for his interest, advice, and material assistance in the preparation of this thesis.

PURPOSE

The purpose of this investigation is: (1) to develop a plan for the effective determination of applicability of the MacAdam ellipses to small color differences in textile samples; (2) to determine the limits of chromaticity perceptibility of the human eye to small color differences in wool textile samples; and (3) to establish a basis for the graphic specification of color tolerances in textile samples.

THEORY

A. Introduction

As a result of the adoption of the standard observer and a set of three primaries by the I.C.I. in 1931, it has been possible to specify colors satisfactorily in this system[1]. This specification alone is not sufficient to denote the least perceptible difference between color A, whose coordinates are x, y, and Y, and color B, whose coordinates are $(x + \Delta x)$, $(y + \Delta y)$, and $(Y + \Delta Y)$.

A survey of previous investigations into the problem has been made by Davidson[2]. He describes the classes of data previously accumulated and points out the need for further data on the sensitivity of the eye to small color differences of surface colors under common viewing conditions. His investigation was intended partially to fulfill that need. This paper, also, is intended to provide data to that end.

Since color can be specified by a series of coordinates, it follows that there should be a means of specifying color tolerances in these coordinates. A means

of color tolerance specification is definitely needed. But if this color tolerance specification is to have any meaning, if it is to have any practical use, it must be based on a sound interpretation of fact. It must not be an arbitrary specification. In order to remove it from the realm of the arbitrary, the sensitivity of the eye, under common viewing conditions, must be considered. Obviously, it is not necessary to set the specifications closer than can be seen by the eye under common viewing conditions. This would necessitate super-control and the attendant increase in cost. On the other hand, it would be folly to set the specification so far removed that it could easily be detected by the eye under common viewing conditions.

The need for a color tolerance specification is as necessary as the need for dimensional specifications in the machine tool industry. It could well be common practice in the future for textile fabric contracts to contain color tolerance specifications, much the same as they are included in other industrial contracts today.

It will be a great satisfaction for purchaser and manufacturer alike to be sure of their ground when

entering into a contract. The manufacturer will have no fear that the fabric he ships will be rejected as "shady goods" or off color, as long as he has assured himself that the color tolerance specification has been met; and, if necessary, he can prove his point. By the same token, the purchaser can rest assured that the fabric he buys will meet the specifications, or it can definitely be rejected with no point for argument.

No longer should it be necessary for the purchaser to depend on the sample of the color he sends to the mill and run the risk of its fading or becoming soiled in handling, to alter the color of the finished product. The specification may be sent by letter, by telegraph, or by telephone with complete assurance. However, until the time when all doubt regarding the color tolerance specification has been removed, the mill will want a sample to work with, in addition to the specification. Subsequent measurements will surely reveal whether or not the "standard" has changed in time. The sample can always be used in getting the desired color within range of the small color difference concept.

The following is a paragraph quoted in part from a paper by MacAdam[3]:

"The measurement and specification of color is a well established technique. Color specifications reveal whether or not two sources radiating different spectral distributions appear to have the same color, for the average human observer. Similarly, the colors of reflecting materials are specified in a manner which indicates immediately whether or not several samples having different spectral reflectance characteristics appear alike under certain conditions of illumination. These specifications give precise meaning to color standards. The specification of color standards can be recorded, and can be communicated, in the form of quantitative values which are reproducible at will in all adequately equipped laboratories. Color standards need no longer be dependent upon the preservation of material samples of questionable permanence."

It must also be noted that until the time when a color tolerance specification is established there will be continued confusion between the terms "perceptibility" and "acceptability". There will be, as well, a tendency toward convenient substitution of one for the other, as the case may require. It is important to note that in theory, perceptibility is the only concept upon which to base a color tolerance specification[4]. However, in practice, acceptability is the guide. This results in a situation where the idiosyncracies of the purchaser must be taken into account in obtaining a "satisfactory color match". In the cases where these prejudices are not known, doubt exists.

Where they are known, it means that a myriad of individual "standards" must be considered. In case it is not possible to obtain a match, the specification may have to be changed to make a pleasing mis-match acceptable.

It is, no doubt, true that the perceptibility limits are smaller than the acceptability limits. Yet it is on the basis of perceptibility limits which we must seek the color tolerance specification, for it will be some fraction of all acceptability standards. It will also define the smallest limit which we need specify. Once the perceptibility limit is established, any size of acceptability limit may be set with full knowledge of what it means.

D. L. MacAdam[5] published a comprehensive study of visual sensitivity to color differences based on a large volume of data. His measurements were made at constant luminosity, enabling him to represent graphically the small color differences as a planar quantity, chromaticity. He concluded from an analysis of these data that an area in the shape of a ellipse about a given point on the chromaticity diagram would include all the chromaticity points which were indistinguishable from the point at the

center of the ellipse. The size of the ellipses, their eccentricities and their orientation varied from region to region on the chromaticity triangle. A graphical representation of twenty-five such ellipses was shown in that paper. Silberstein[6], in a theoretical study, confirmed the fact that the data should be elliptical when plotted.

At a later date, MacAdam[7] published a paper which developed the idea that there was a system to the ellipses, as might be inferred from the graphical representation of the twenty-five ellipses on the chromaticity diagram. As a result of that work, it is no longer necessary to extrapolate graphically among the twenty-five ellipses to find the size and shape of the ellipse at any desired point on the chromaticity diagram.

By means of three contour-like diagrams, it is possible to obtain three constants which, when substituted into the following equations, yield: (1) the length of the semi-major axis, (2) the length of the semi-minor axis, and (3) the angle of inclination of the major axis of the ellipse to the horizontal for any desired point on the chromaticity diagram.

$$(1) \quad a^2 = \frac{1}{g_{22} + g_{12} \quad \text{Cot } \theta}$$

$$(2) \quad b^2 = \frac{1}{g_{11} - g_{12} \quad \text{Cot } \theta}$$

$$(3) \quad \tan 2\theta = \frac{2g_{12}}{g_{11} - g_{22}}$$

This might at first appear to be a complete solution to the problem. On the basis of these data and upon the conclusions drawn from it, we should have a true color tolerance specification. However, it must be remembered that this is only the first step, that these measurements were made under ideal viewing conditions, that beams of light were used for comparison, an extremely limited application, and not surface reflecting samples. Further, a small field (2°) was used, and Wright states that discrimination is inversely proportional to field size. In other words, the smaller the field, the larger the detectable difference. However, it should be proportional throughout the spectrum.

What remains to be done is to apply, if possible, MacAdam's data to other fields, particularly to the field of surface reflecting materials, and specifically to surface reflecting textile materials.

In still another paper by Brown and MacAdam[8], the investigation was carried to the three dimensional aspect of the visual sensitivity to combined chromaticity and luminosity differences. Eventually, the problem at hand must be considered in this respect, but in this investigation in order to use MacAdam's data, restricting the variations to the plane of the chromaticity diagram was necessary.

B. Statement of the Problem

The problem stated simply is this: Can the visual sensitivity to chromaticity as established by the MacAdam papers be used in practical applications to the color difference problems occurring in wool dyeings and blends?

In order to answer this question, an experiment must be devised to supply measurement data sufficient in scope to support a general conclusion. Certain steps are required to conduct the experiment to the desired end. These are: (1) to obtain a supply of samples, (2) to have a means of measuring the samples, (3) to provide a means of visually judging the samples, and (4) to make an interpretation of the data.

As regards the first step, the supply of samples, there should be a sufficient number to approach the problem with some statistical surety. They should be made easily and quickly. The tests, in effect, are "destructive" since the samples must be small enough to be inserted in the spectrophotometer and also small enough to be handled in quantity by the judging colorists. There must be a means of varying the chromaticity as desired and holding

the luminosity constant. They should be fast to light. They should be self shades, close enough to one another so that the difference in color can be classed as a small color difference.

As regards the second step, a means of measuring the samples, first consideration must be given to the system of coordinates best suited. Fortunately, the system of coordinates which is most adaptable is that of the International Commission on Illumination. Since a great deal of the basic work has been done in this system, together with the fact that the recording spectrophotometer is a most acceptable measuring device which produces answers in this system, the I.C.I. system and the recording spectrophotometer are well suited as the means of measuring and designating the colors.

As regards the third step, a means of visually judging the samples, a "standard observer" is needed whose judgments are completely objective, but the "standard observer" is unavailable. Therefore, some approximation to the "standard observer" must be made. A group of colorists should be available to judge the samples. They should judge the samples in such a way as to minimize

personal prejudices. The viewing conditions should be standardized. Each sample should be judged a number of times and the sequence should be randomized.

As regards the fourth step, an interpretation of data obtained under steps two and three, a correlation or lack of correlation between the two sets of data must be sought. This correlation must be approached from a statistical viewpoint.

C. Proposed Method of Solution

A satisfactory supply of samples may be obtained by padding worsted top. In this manner any desired number may be made quickly and easily. No difficult is encountered due to the "destructive" nature of the requirements because the samples are made for the purpose of the tests only. The top may be dyed to any color whose region on the chromaticity diagram it is desired to investigate, bearing in mind the fact that present known dyestuffs do not cover the chromaticity diagram completely. Self shade dyeings may be made, of which any blend desired may be made. This gives considerable latitude in selecting samples of almost any desired chromaticity. And it is assumed for the present that the samples so prepared will be of very nearly equal luminosity. This method seems to fulfill the requirements as set forth in the statement of the problem, that the color differences obtained may be classed as small color differences.

The means of measuring the samples and designating them in the I.C.I. system is adequately provided in the General Electric recording spectrophotometer[9], equipped with a GAF librascope tristimulus integrator[10].

It poses no real problem for it is relatively simple to operate. The measurement cycle is started and when it is finished the tristimulus values X, Y, and Z are read directly from the integrator. The brightness or luminosity value is given directly as Y. The trichromatic coefficients x and y are obtained by simple calculations from the following relationships: $x = \frac{X}{X + Y + Z}$ and $y = \frac{Y}{X + Y + Z}$. All that remains to complete this part of the experiment is to plot the chromaticity values on an expanded (large scale) portion of the chromaticity diagram.

The means of judging the samples is relatively more difficult in that it requires the time of the colorists and a recorder to determine and tabulate the data. A system similar to the one suggested by Davidson[2] seems satisfactory. It involves having the colorists judge each sample relative to that sample arbitrarily designated as the standard. Each sample is judged three times non-consecutively and at random. The colorist is asked to say whether the sample in question is regarded by him as brighter, duller or a match in brightness; redder, greener, bluer, yellower, or a match in hue; weaker,

stronger, or a match in strength. This method follows closely the common practice among commercial textile colorists. A point in question is whether or not these factors--brightness, hue, and strength--are the counterparts of the I.C.I. system factors--luminance, dominant wave length, and purity. The difference is a subtle one, being that the former are sensations which cannot be measured and the latter three are called psychophysical measurements which can be obtained by means of the spectrophotometer and the "standard observer". For the purposes of this work, it is assumed that there is a correlation among the terms. In further writing this point will be discussed in more detail.

Five or more individual colorists should be asked to contribute this information. It must be collected and analyzed statistically in order to arrive at a single evaluation for each sample. The statistical consensus is taken to be the decision of the "standard observer".

The interpretation phase of the experiment consists of plotting the chromaticity of each sample on the chromaticity diagram, constructing the MacAdam ellipse for the chromaticity point of the standard from values

obtained from the equations described earlier. This divides the points into two groups--those lying inside and those lying outside of the ellipse. The standard observer's decision also will have divided the data into two groups--those samples which match the standard and those which do not.

This gives rise to four possible situations. First, that all the points inside the ellipse are judged to be a match in chromaticity and all the points outside the ellipse are judged to be not a match in chromaticity. Second, that all points inside the ellipse and some outside the ellipse are judged to be a match. Third, that all points outside and some inside are judged to be not a match. Fourth, that there is an indiscriminate distribution of matches and non-matches, that is, some points inside and some outside are judged to be a match while some points inside and some outside are judged to be not a match.

If the first case is found to be the fact, then there is some correlation and the MacAdam ellipse as calculated may be used as the limit of perceptibility for textile samples made from wool. If case two is found to be

the fact, the ellipse may have to be enlarged. If case three is found to be the fact, the ellipse may have to be made smaller. Or if case fouris found to be the fact, then there may be no correlation and the ellipse theory may have to be set aside for this particular purpose.

D. Difficulties Encountered

Since very little work has been published on this specific problem to date, the first experiments must necessarily be in the form of limited feasibility tests. Therefore, it was decided to limit the scope of the experiment to a study of textile samples made of wool. The type of samples described herein have several advantages over types used in previous experiments. The dyed swatches used by Davidson[2] seem to be more difficult to obtain in quantity and in the desired chromaticity. Variables in dyeing should prove to be a hindrance. The samples used by Scott[11] were taken, as available, from the files of an inspection department. There was no choice as to number of samples nor as to the chromaticity. The permanence of characteristics of these samples is questionable due to the length of time they may have been in the files. See Figure VII.

The proposed method is not without its limitations, however, For example, the assumption that two self shades of 0.75% and 1% dye on the weight of stock would give a satisfactory spread of chromaticities was disproved because the points occurred too far apart on the

chromaticity diagram. The remedy, of course, was to make the dye concentrations closer together.

Even after the dye strengths were brought closer together, it was found that pads made entirely of only one shade of the dye were giving a considerable distribution of chromaticities. There was no known way of controlling these chromaticities and they had to be accepted as they were, since the pad making procedure had been standardized as far as possible.

Regarding blends of the shades, it was thought that the locus of chromaticity points should proceed in a fairly straight line from 100% shade A to 100% shade B, a relatively short distance, depending upon the percentages of each shade used for a particular sample. This proved to be the case for the first series of orange pads. However, when it was tried to obtain chromaticity points in between two points already plotted, those points positioned themselves below the original locus of points and were out of the sequence which might be expected from the percentages of each shade making up the samples. See Figure V.

The dyed stock was inadvertently left exposed to light in the laboratory between the first and second

series of pads. It is believed that the stock faded during this time, accounting for the displacement of the points below the original locus of points. The variance due to the pad making may explain why the points occurred out of sequence. It is possible that one pad obtained the maximum variance to the right while the adjacent one obtained the maximum variance to the left, and as a result they were apparently interchanged in position. None of the orange samples were put to the colorists' test phase of the experiment because there were not enough samples in the first series whose chromaticities were in the most desirable positions relative to the standard and, therefore, relative to the MacAdam ellipse constructed around it. The obvious fading of the stock precluded any confidence in results subsequently obtained. It was, therefore, decided to set aside this work in order to incorporate the intelligence obtained into the next series of samples of another color. See Figure VI for construction of ellipse and plot of points.

PROCEDURE

A. Making the Samples

Raw material:	64's Worsted Top
Weight of top each dyeing:	25 grams
Dye:	(1) Milling Fast Orange, 0.75% and 1.0% Strength (2) Palatine Fast Blue GGN, 0.75% and 0.90% Strength
Weight of each sample:	1.5 grams total
Blending:	Hand Card
Fulling Solution:	37.5 grams Colgate's #10 soap and 15.0 grams of soda ash per liter of water
Padding Machine:	See sketch following
Padding time:	3 - 6 minutes per side - Average time 4 minutes per side - Use 10 cc of fulling solution per pad, plus a small amount of hot water
Washing and drying:	Washed with cool water by hand - pressed excess moisture out by hand in towel - Dried completely on a hot head press set to the temperature for wool
Composition of each sample:	See chart following and Figure I.

The worsted top was washed in Naccanol. One 25 gram lot was dyed with 0.75% of orange dye on the weight of

stock and another 25 grams with 1.0% orange dye. The same procedure was followed throughout for the blue samples except that the dye strengths were 0.75% and 0.90%.

When dyeing was completed and the stock was dry, it was found to have been slightly felted or matted, due to the agitation in the hot dye bath. The hand card was used to open and disentangle the fibers. The desired amount of each of the two shades of orange was weighed out on a beam balance and blended on the hand card. Small tufts of each shade were interspersed to start the blending, followed by successive carding cycles until the fibers became a homogeneous mass.

This blend of fibers was placed in the padding machine together with 10 cc of the fulling soap described above. The top weight was inserted and the machine was turned on. Enough hot water was added to keep the weight rotating slowly in the opposite direction to that of the eccentric.

At the end of four minutes the machine was stopped, the pad was turned over and run again for four minutes. The pad was then removed, washed gently under the tap, pressed free of excess water with a towel, and hot

pressed until it was completely dry. Each sample was marked and inserted in an evelope to guard against fading.

Composition of Samples

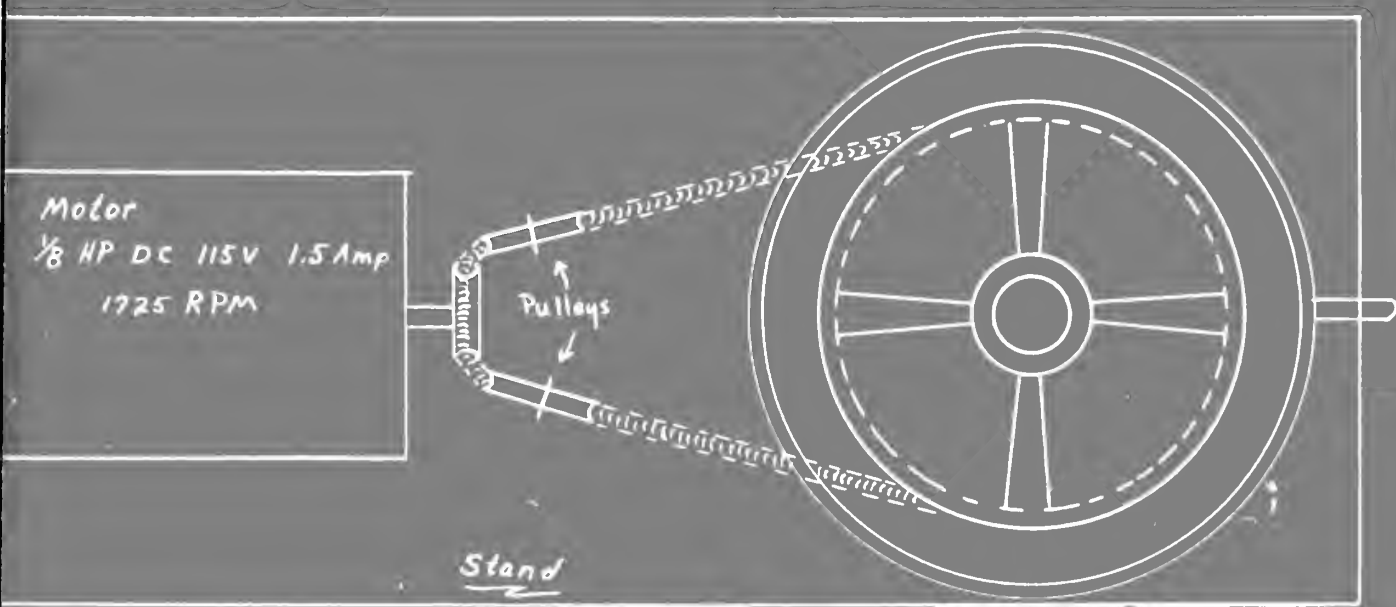
Milling Fast Orange

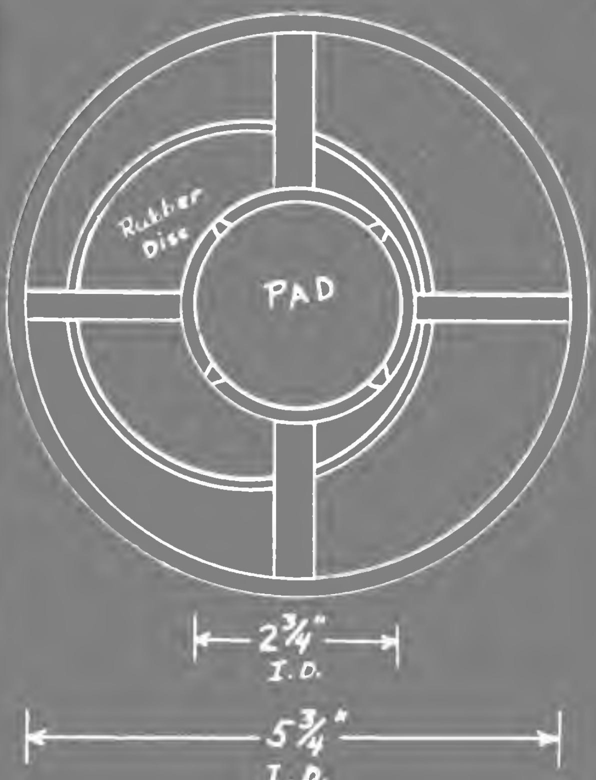
	Weight		Percentage	
	0.75% Strength	1.0% Strength	0.75% Strength	1.0% Strength
1	1.5 gms	0 gms	100 %	0 %
2	1.2	0.3	80	20
3	0.9	0.6	60	40
3A	0.875	0.625	58.3	41.7
3B	0.85	0.65	56.6	43.4
3C	0.8	0.7	53.3	46.7
4	0.75	0.75	50	50
5	0.6	0.9	40	60
5A	0.5	1.0	33.3	66.7
5B	0.4	1.1	26.7	73.3
5C	0.35	1.15	23.3	76.7
6	0.3	1.2	20	80
7	0	1.5	0	100

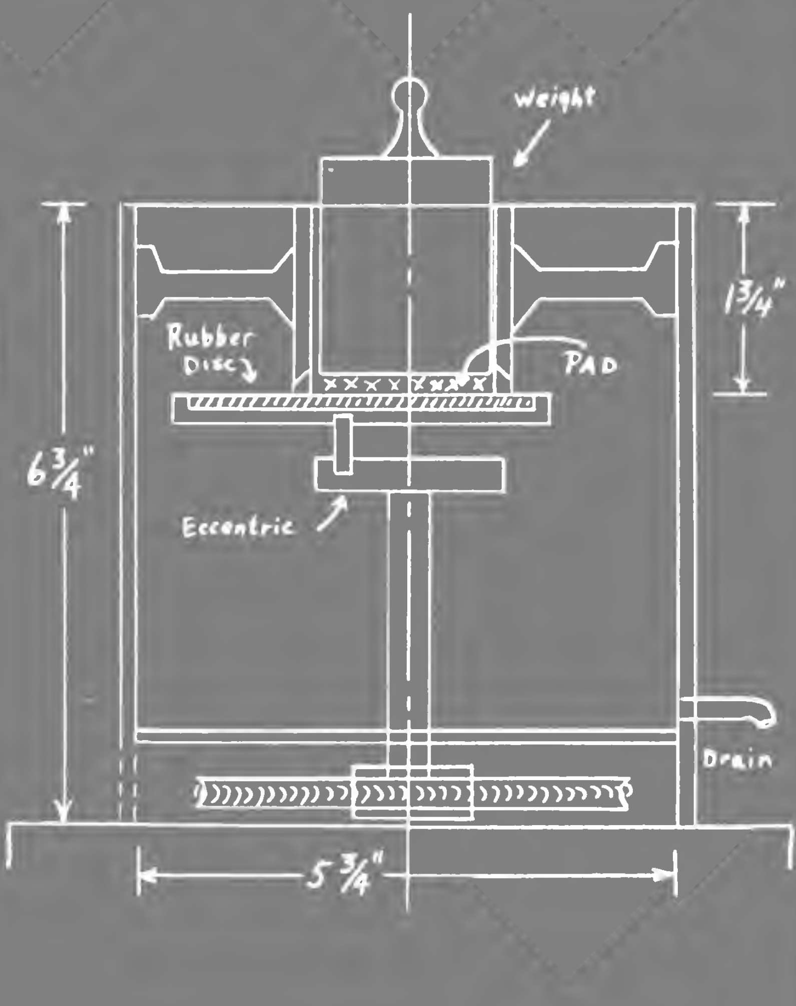
Palatine Fast Blue GGN

	Weight		Percentage	
	0.75% Strength	0.9% Strength	0.75% Strength	0.9% Strength
1	1.5 gms	0 gms	100%	0%
2	1.2	0.3	80	20
3	0.9	0.6	60	40
4	0.75	0.75	50	50
5	0.6	0.9	40	60
6	0.3	1.2	20	80
7	0.0	1.5	0	100
8	0.75	0.75	50	50
9	0.75	0.75	50	50
10	0.75	0.75	50	50
11	0.75	0.75	50	50
12	0.75	0.75	50	50
13	0.75	0.75	50	50
14	1.5	0	100	0
15	1.5	0	100	0
16	1.5	0	100	0
17	1.5	0	100	0
18	1.5	0	100	0
19	1.5	0	100	0
20	0	1.5	0	100
21	0	1.5	0	100
22	0	1.5	0	100
23	0	1.5	0	100
24	0	1.5	0	100
25	0	1.5	0	100

Padding Machine







B. Measuring the Samples

Instrument:	General Electric Recording Spectrophotometer with GAF librascope tristimulus integrator located at the Darby Company, Inc. Laboratory, 49 Blanchard Street, Lawrence, Massachusetts
Description:	See Bibliography - (12)
Standard:	White vitrolite
Precautions:	Each sample was backed with another to make sure no light was transmitted through the sample - Integrating sphere was covered
R.S.P. Curves:	See sample curves following
Calibration[12]:	The curve of a neutral Jena filter was taken at the beginning of each series of measurements, to be sure that the instrument was calibrated the same on each occasion
Integrator:	The variance of the integrator was calculated from twenty successive measurements on the same sample - the results are shown below
Calculations:	(A) The tristimulus values X, Y, and Z were read directly from the integrator at the completion of the measurement cycle. The trichromatic coefficients x and y were calculated from the following formulas:

$$x = \frac{X}{X + Y + Z} \qquad y = \frac{Y}{X + Y + Z}$$

These values were then plotted on a large scale chromaticity diagram.

(B) Method of plotting the MacAdam ellipses for any chromaticity at equal luminance:

(1) Find x and y of the standard.

(2) Using the appropriate charts[5], read values of g_{11}, g_{22}, and $2g_{12}$.

(3) From the following equations, compute:

(a) θ, the angle the major axis makes with the horizontal

(b) a, the semi-major axis

(c) b, the semi-minor axis

Equations:

(a) $$\operatorname{Tan} 2\theta = \frac{2g_{12}}{g_{11} - g_{22}}$$

or $$2\theta = \operatorname{Tan}^{-1} \frac{2g_{12}}{g_{11} - g_{22}}$$

(b) $$a^2 = \frac{1}{g_{22} + (g_{12} \times g_{12} \operatorname{Cot}\theta)}$$

(c) $$b^2 = \frac{1}{g_{11} - g_{12} \operatorname{Cot}\theta}$$

Note: These equations apply only to the case where the luminosity or brightness difference between the standard and the match are insignificant. This holds for the resulting ellipses also.

The semi-minor and semi-major axes, a and b, should be multiplied by [illegible] to obtain the axes of an ellipse pertaining to a just noticeable difference (j.n.d.). In finding the angle 2θ, it should be remembered that if $2\theta = \mathrm{Tan}^{-1} A$, then if A is minus, $2\theta = 180^{\circ} - 2\theta$.

Sample Calculation:

Blue Standard, Sample #4

$x = .2160$

$y = .2267$

From Charts:

$g_{11} = 133 \times 10^4$

$2g_{12} = -70 \times 10^4$

$g_{12} = -35 \times 10^4$

$g_{22} = 28 \times 10^4$

$$\mathrm{Tan}\ 2\theta = \frac{(-70 \times 10^4)}{(133 \times 10^4) - (28 \times 10^4)} = \frac{-70}{105} = -.666$$

$$2\theta = 180^{\circ} - 2\theta = 180^{\circ} - 33.6^{\circ} = 146.4^{\circ}$$

$$\theta = 73.2\ ,\ \mathrm{Cot}\ \theta = .30192$$

$$a^2 = \frac{1}{28x10^4 + (-35x10^4 x Cot\theta)} = \frac{1}{28 + (-10.63)} x10^{-4} = \frac{1}{17.37} x10^{-4}$$

$$= .0575 \times 10^{-4}$$

$$a = .240 \times 10^{-2} = .0024$$

$$2a = .0048$$

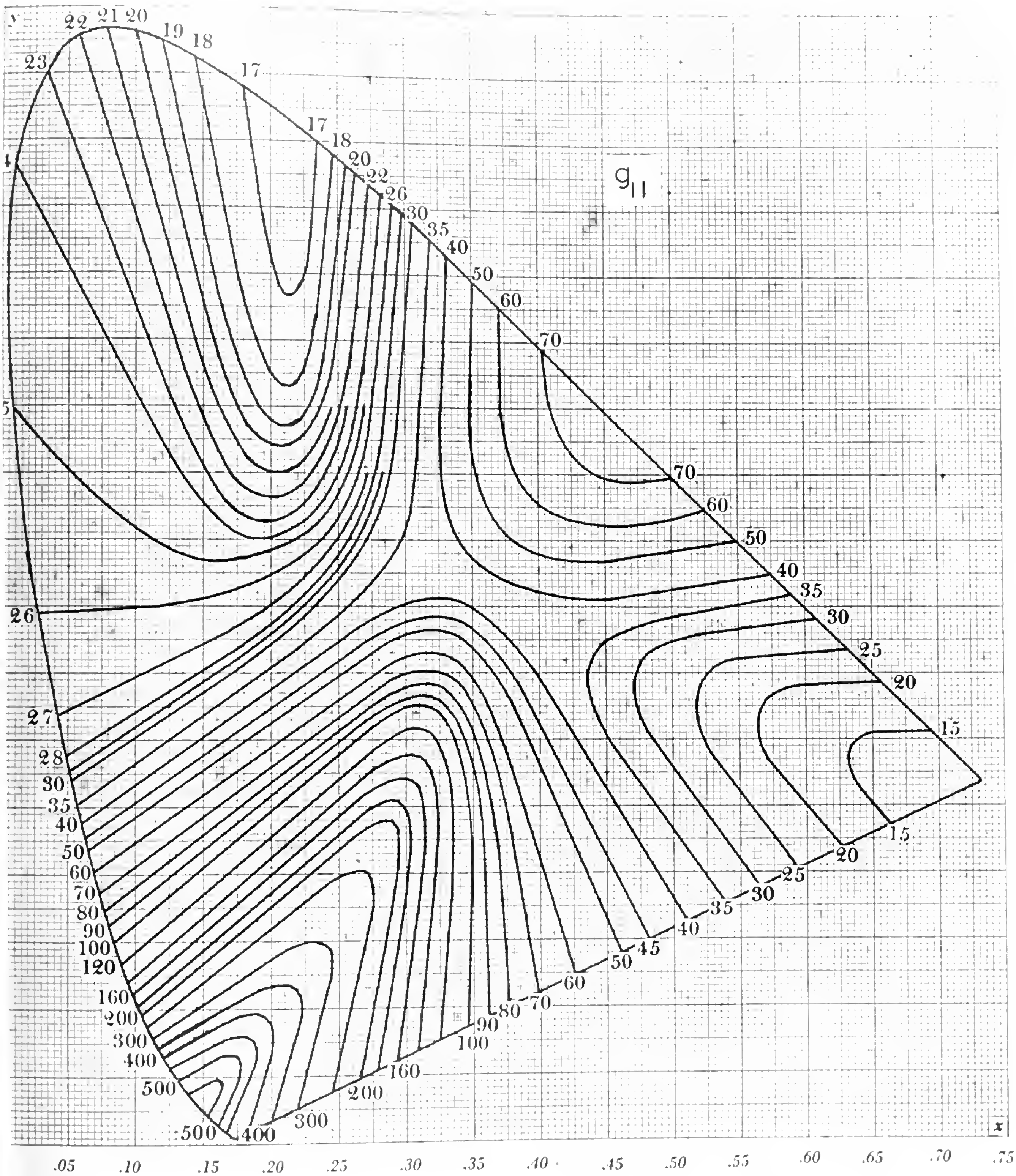
$2a \times 3 = .01440$ $\qquad$ $a \times 3 = .0072$

$$b^2 = \frac{1}{133x10^4 - (-10.63 \times 10^4)} = \frac{1}{143.63} x10^4 = .0069 \times 10^{-4}$$

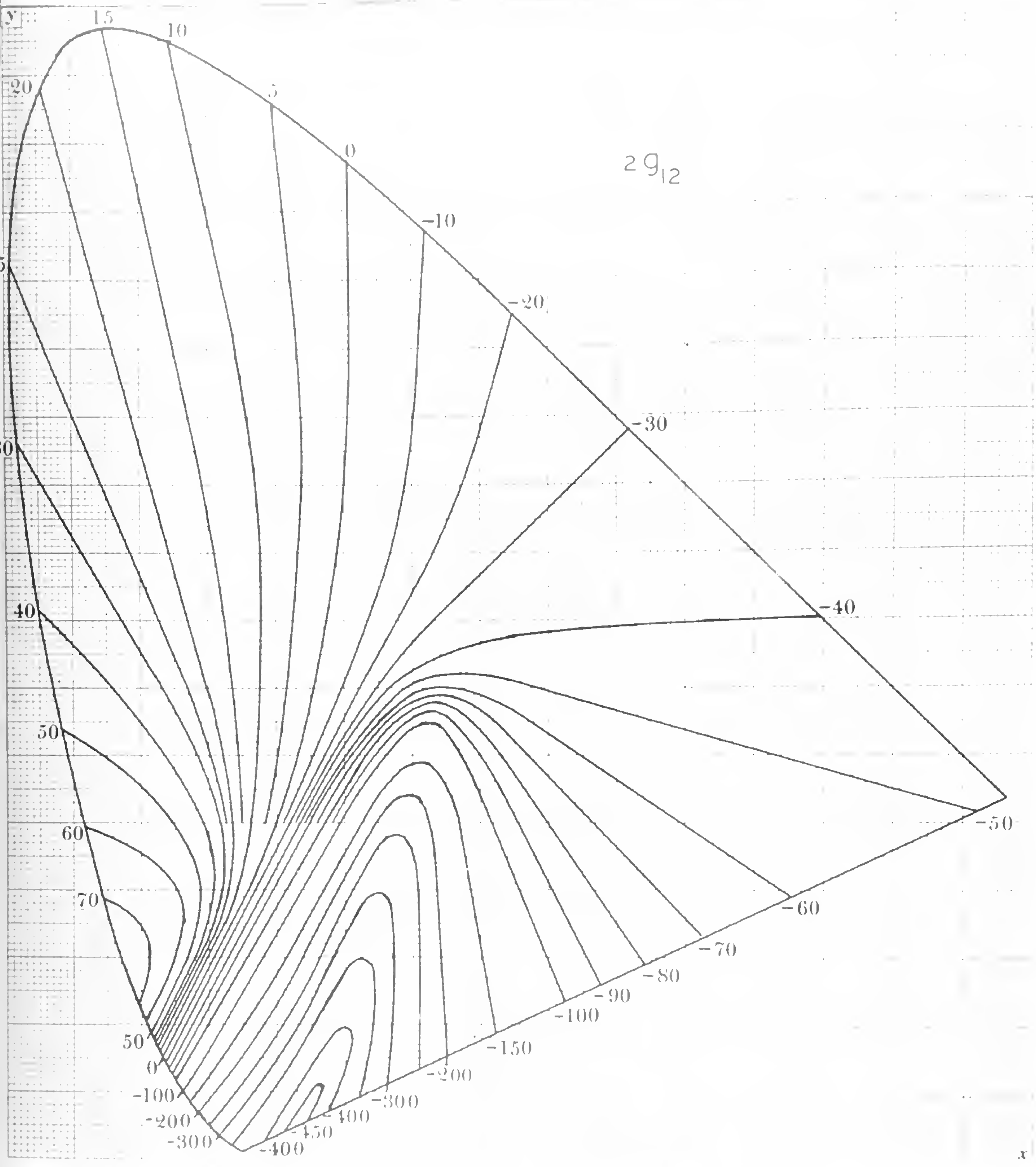
$$b = .0264 \times 10^{-2} = .000264$$

$$2b = .000528$$

$2b \times 3 = .001584$ $\qquad$ $b \times 3 = .000792$

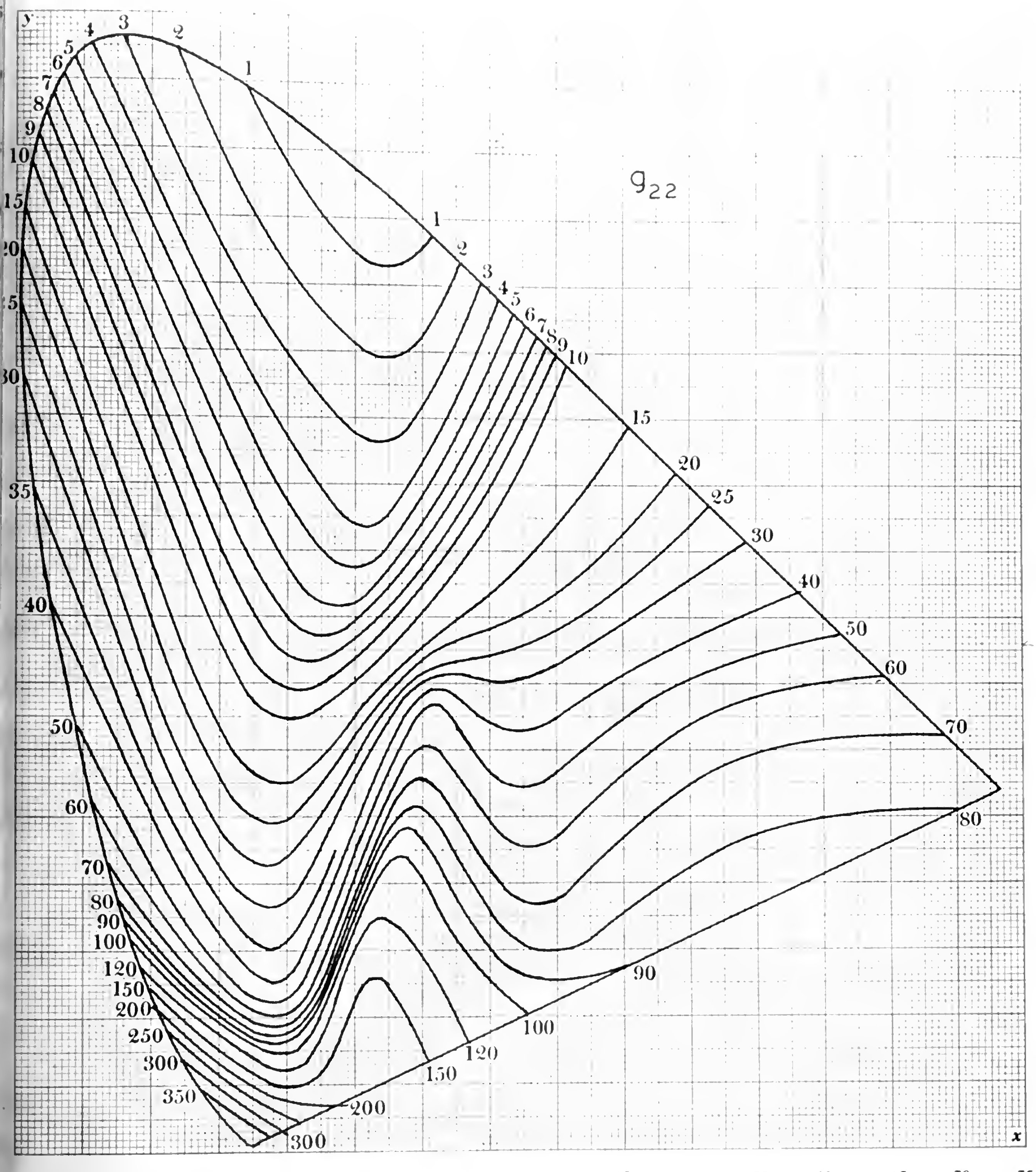


. Values of the coefficient g_{11} to be multiplied by 10^6 for various locations in the I.C.I. chromaticity diagram



y
15
10
20
5
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-150
-200
-300
-400
-450
400
30
40
50
60
70
50
0
-100
-200
-300
x
0
.05
.10
.15
.20
.25
.60
.65
.70
.75

2g12



3. Values of the coefficient g_{22} (to be multiplied by 10^4) for various locations in the I.C.I. chromaticity diagram.

VARIANCE IN THE INTEGRATOR

Deviations from Assumed Mean

	$(x-.2170)$	$(x-.2170)^2$	$(y-.2290)$	$(y-.2290)^2$	$(Y-.0770)$	$(Y-.0770)^2$
1	1	1	2	4	2	4
2	4	16	0	0	0	0
3	1	1	6	36	2	4
4	0	0	10	100	4	16
5	1	1	3	9	1	1
6	2	4	16	256	6	36
7	3	9	5	25	2	4
8	7	49	6	36	2	4
9	12	144	5	25	3	9
10	9	81	11	121	3	9
11	9	81	8	64	2	4
12	19	361	~~65~~	~~4225~~	~~46~~	~~2116~~
13	6	36	2	4	-1	1
14	3	9	3	9	0	0
15	12	144	8	64	2	4
16	6	36	5	25	0	0
17	12	144	11	121	2	4
18	16	256	1	1	-2	4
19	20	400	7	49	2	4
20	~~34~~	~~1156~~	17	289	10	100
Σx & Σx^2	177	2929	191	5463	86	2324
$(\Sigma x)^2$	31329		36481		7396	
Σx & Σx^2	143	1773	126	1238	40	208
$(\Sigma x)^2$	20449		15876		1600	

Q Test: N = 20 $Q = \frac{\text{Difference}}{\text{Range}}$

s_x	Maximum Value #20	34	$Q = \frac{14}{34} = .412$	This exceeds the rejection quotient from the table for N = 20 so the value can be rejected.
	Nearest Value	20		
	Difference	14		
	Range	34		
s_y	Maximum Value #12	65	$Q = \frac{48}{65} = .740$	Ditto
	Nearest Value	17		
	Difference	48		
	Range	65		
s_Y	Maximum Value #12	46	$Q = \frac{36}{46} = .782$	Ditto
	Nearest Value	10		
	Difference	36		
	Range	46		

$$s = \sqrt{\frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{(N)}} = \sqrt{\frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{(N)^2}} \qquad = s\sqrt{\frac{N}{N-1}} \qquad N = 19$$

$$s_x = \sqrt{\frac{1773}{19} - \left(\frac{(143)}{(19)}\right)^2} \qquad s_y = \sqrt{\frac{1238}{19} - \left(\frac{(126)}{(19)}\right)^2} \qquad s_Y = \sqrt{\frac{208}{19} - \left(\frac{(40)}{(19)}\right)^2}$$

$$= \sqrt{\frac{1773}{19} - \frac{20,449}{361}} \qquad = \sqrt{\frac{1238}{19} - \frac{15,876}{361}} \qquad = \sqrt{\frac{208}{19} - \frac{1600}{361}}$$

$$= \sqrt{93.31 - 56.64} \qquad = \sqrt{65.16 - 44} \qquad = \sqrt{10.94 - 4.43}$$

$$= \sqrt{36.67} \qquad = \sqrt{21.16} \qquad = \sqrt{6.51}$$

$$s_x = 6.05 \qquad s_y = 4.59 \qquad s_Y = 2.55$$

.0006 .0005 .0003

Note: (1) The fourth decimal place is estimated when reading the trichromatic coefficients from the integrator.

(2) The smallest scale division on our chromaticity diagrams is 0.0001.

COLOR MEASUREMENT DATA
(ICI Method of Specification)

Date 3/1/51 Sample Identification Blue (5

Illuminant C

(A) TRISTIMULUS VALUES

X = .0734

Y = .0769

Z = .1886

X + Y + Z = .3389

(B) TRICHROMATIC COEFFICIENTS

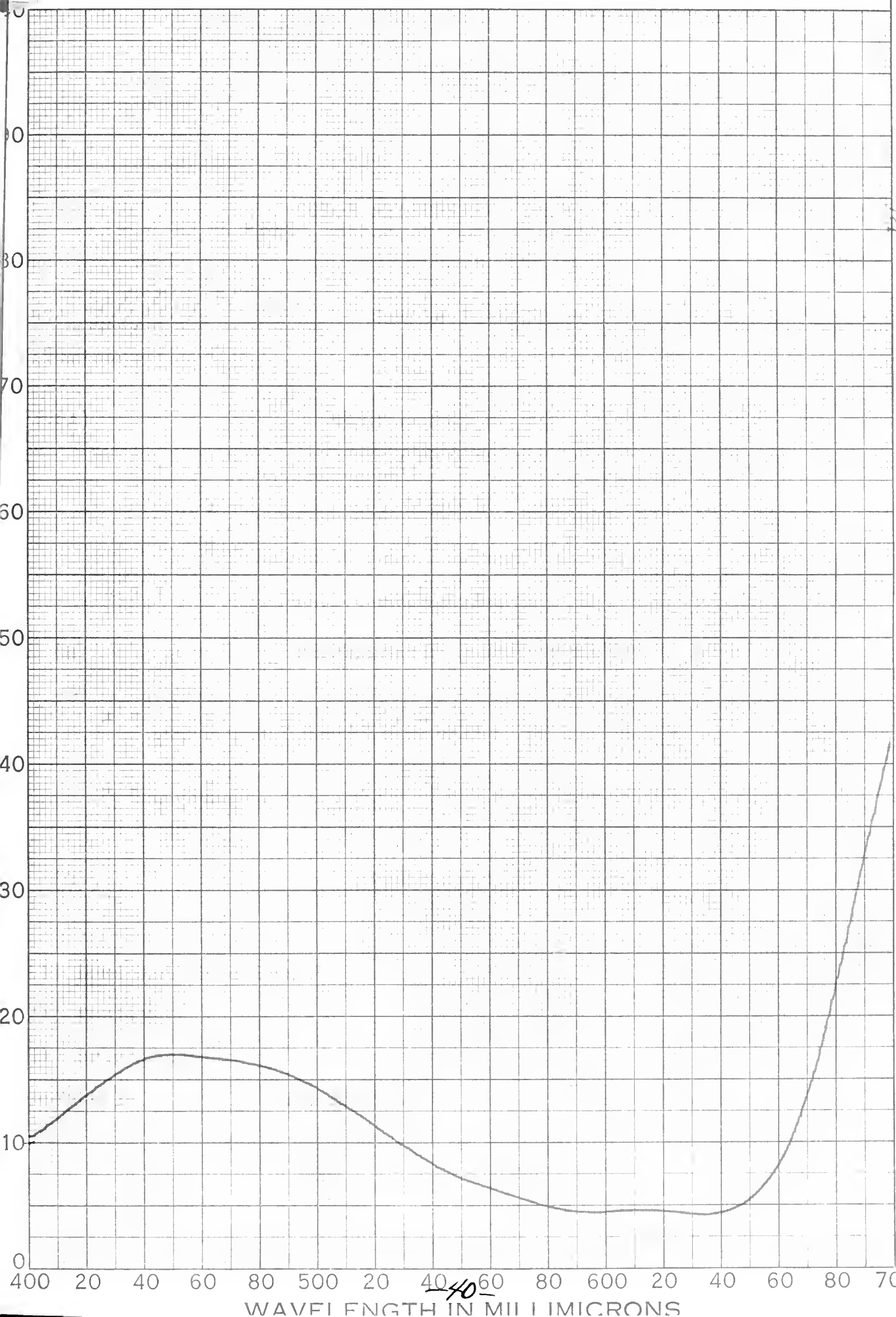
$x = \frac{X}{X + Y + Z} =$ .2165

$x = \frac{Y}{X + Y + Z} =$ .2269

(C) SPECIFICATIONS

	Sample	Standard
(1) Dominant Wave Length		
(2) Purity		
(3) Brightness		

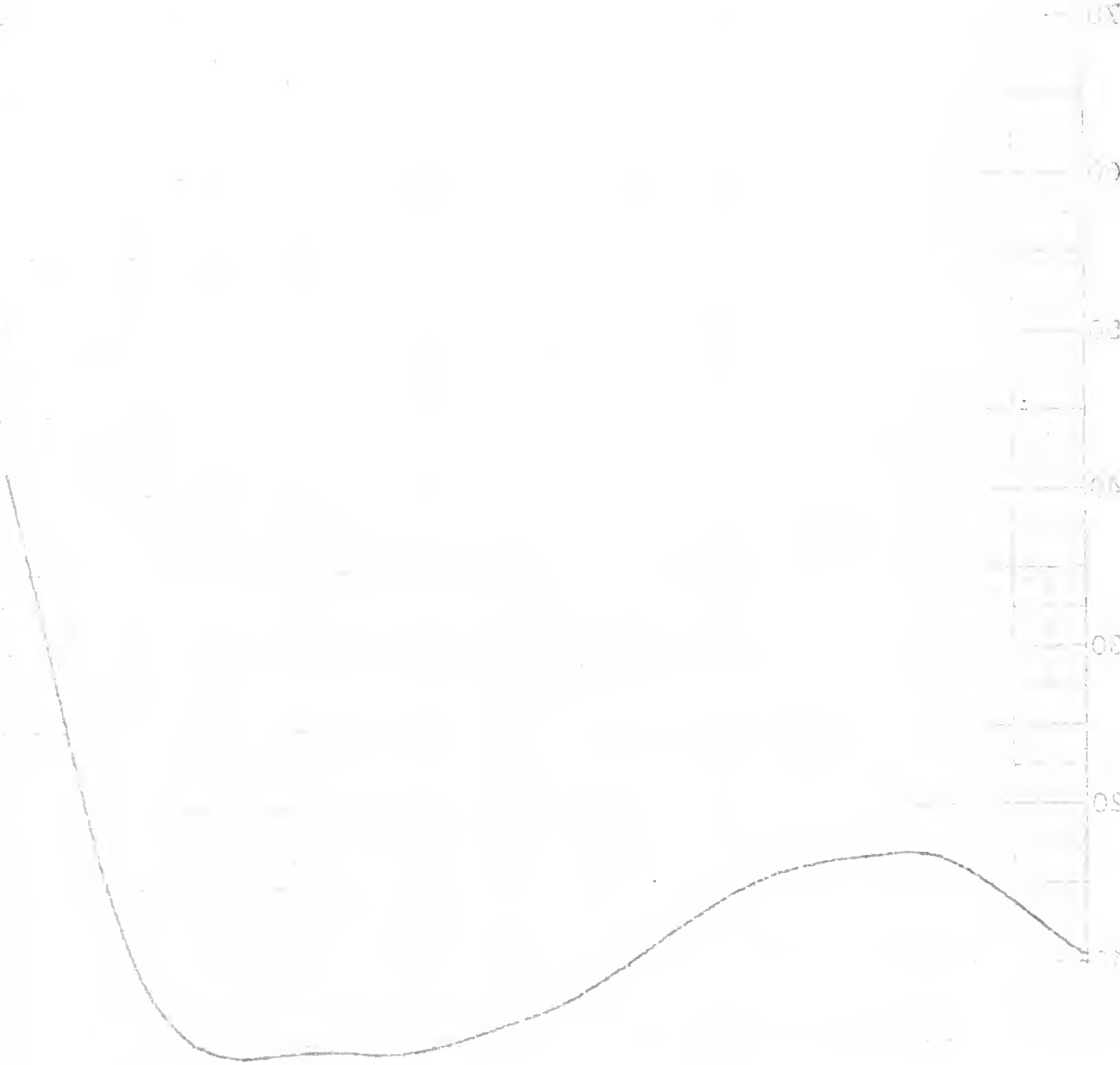
Acceptable ______ Not Acceptable ______ Border Line ______



TEST NO. 5 BY
DATE

GENERAL ELECTRIC

INSTRUMENT NO.



COLOR MEASUREMENT DATA
(ICI Method of Specification)

Date 3/1/51 Sample Identification Blue C

Illuminant C

(A) TRISTIMULUS VALUES

X = .0662

Y = .0684

Z = .1736

X + Y + Z = .3082

(B) TRICHROMATIC COEFFICIENTS

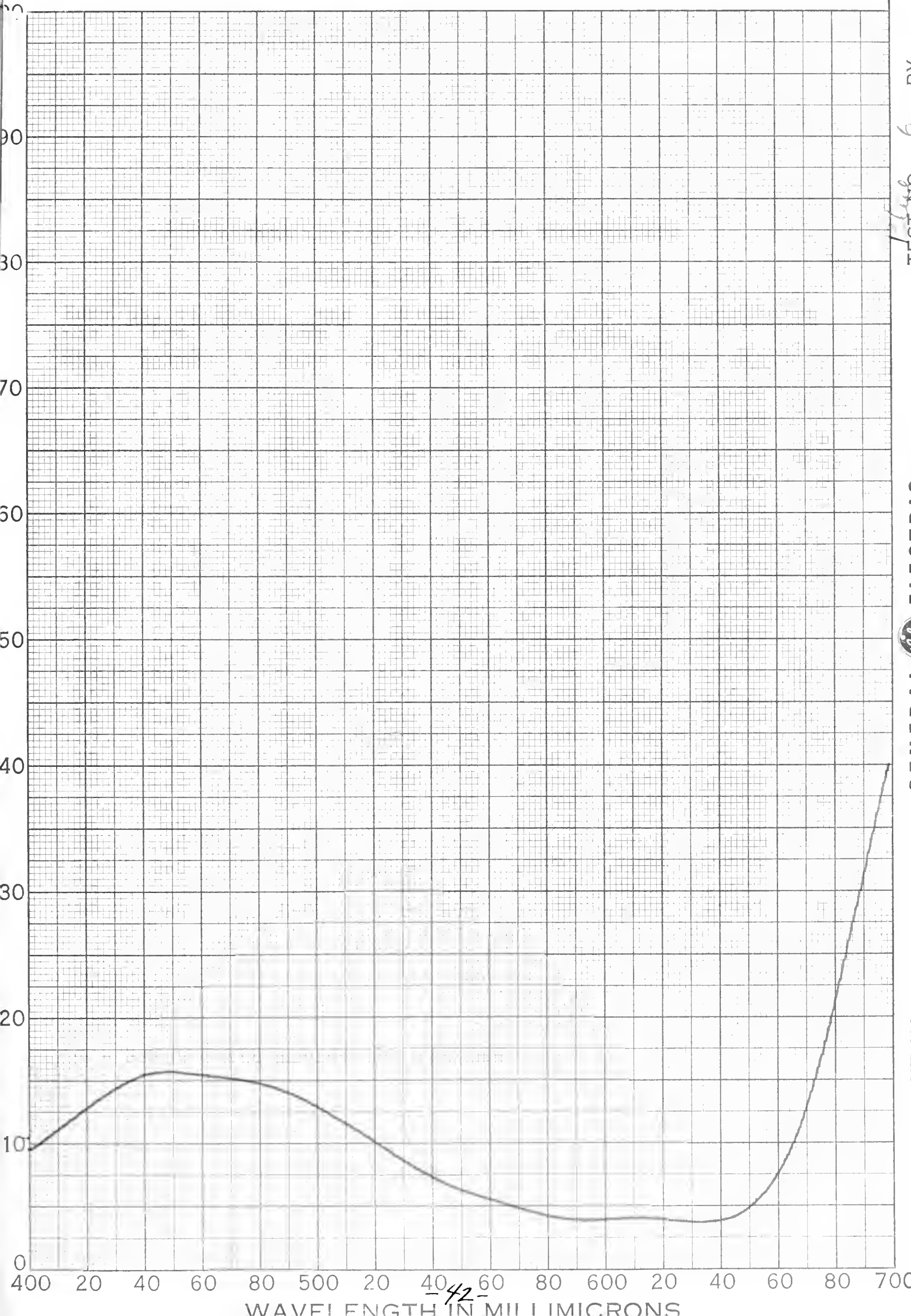
$$x = \frac{X}{X + Y + Z} = .2147$$

$$x = \frac{Y}{X + Y + Z} = .2219$$

(C) SPECIFICATIONS

	Sample	Standard
(1) Dominant Wave Length		
(2) Purity		
(3) Brightness		

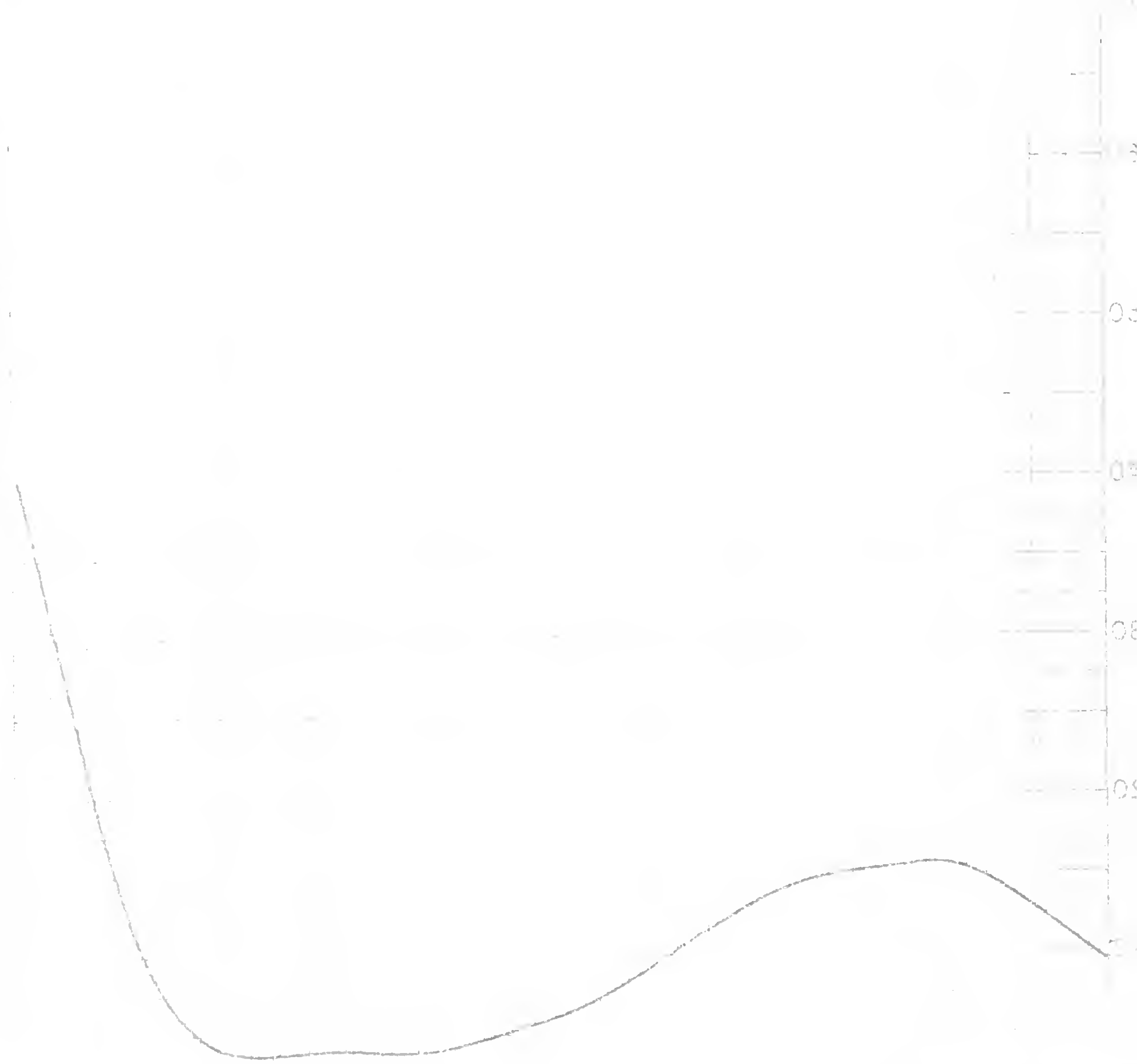
Acceptable ______ Not Acceptable ______ Border Line ______



TEST NO. 6 BY
DATE

GENERAL ELECTRIC

INSTRUMENT NO.

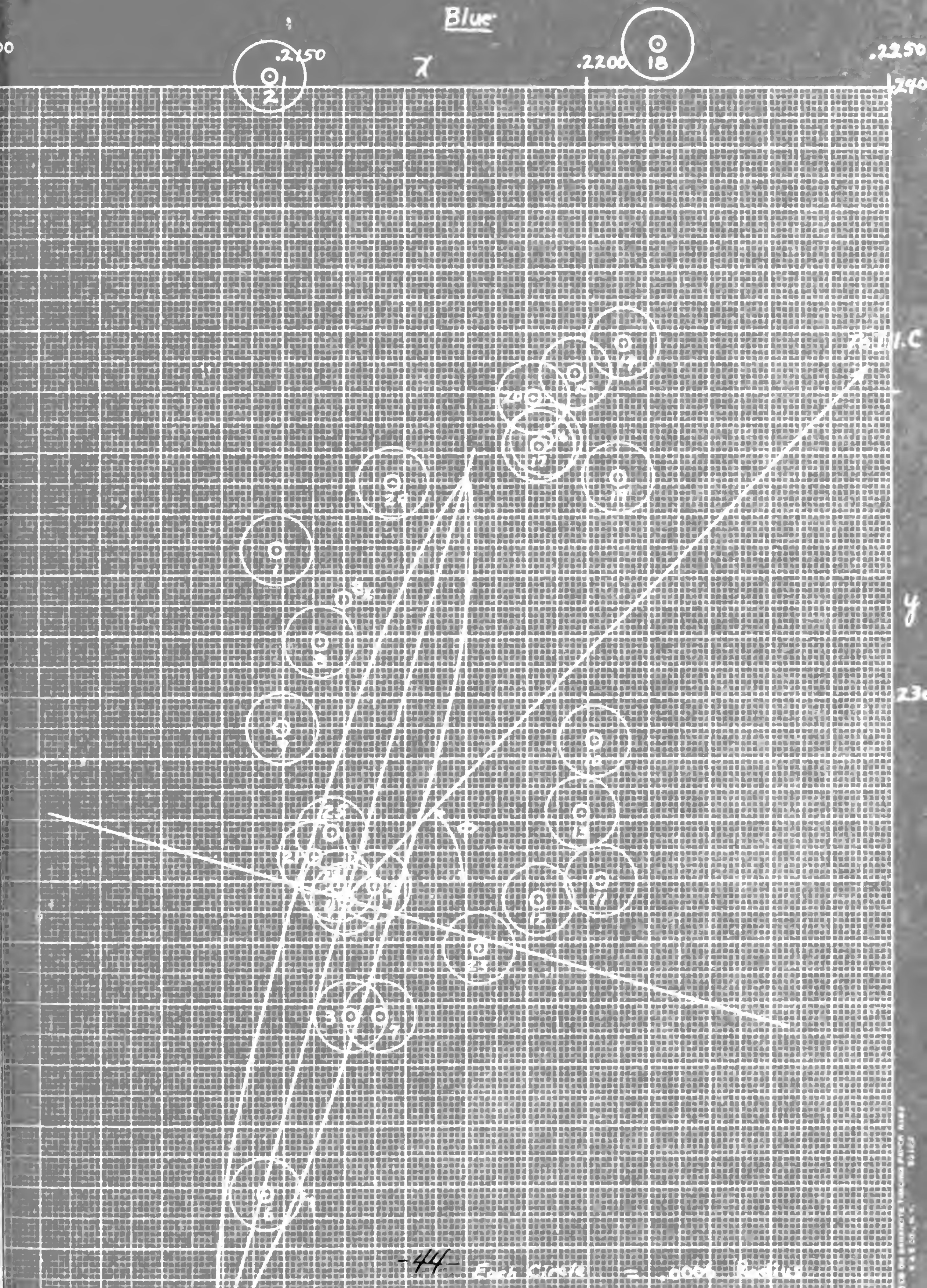


SAMPLE COMPOSITION VS. I.C.I. COORDINATES

Palatine Fast Blue GGN

Sample No.	Wt. of Stock - gms Lye 0.75%	Lye 0.9%	Blend Percent 0.75%	0.9%	Chromaticity Points x	y	Brightness Y	
1	1.5	0.0	100	0	.2149	.2324	.0773	
2	1.2	0.3	80	20	.2148	.2402	.0853	
3	0.9	0.6	60	40	.2161	.2248	.0753	*
4	0.75	0.75	50	50	.2160	.2267	.0737	Std.
5	0.6	0.9	40	60	.2165	.2269	.0769	*
6	0.3	1.2	20	80	.2147	.2219	.0684	*
7	0.0	1.5	100	0	.2166	.2248	.0719	
8	0.75	0.75	50	50	.2156	.2309	.0814	
9	0.75	0.75	50	50	.2150	.2295	.0776	
10	0.75	0.75	50	50	.2201	.2293	.0773	
11	0.75	0.75	50	50	.2202	.2270	.0762	
12	0.75	0.75	50	50	.2192	.2267	.0756	
13	0.75	0.75	50	50	.2199	.2281	.0783	
14	1.5	0.0	100	0	.2206	.2358	.0865	
15	1.5	0.0	100	0	.2198	.2353	.0862	
16	1.5	0.0	100	0	.2193	.2342	.0865	
17	1.5	0.0	100	0	.2192	.2341	.0832	
18	1.5	0.0	100	0	.2212	.2408	.0920	
19	1.5	0.0	100	0	.2205	.2336	.0839	
20	0.0	1.5	0	100	.2191	.2349	.0818	*
21	0.0	1.5	0	100	.2155	.2274	.0712	*
22	0.0	1.5	0	100	.2159	.2269	.0720	
23	0.0	1.5	0	100	.2182	.2259	.0727	
24	0.0	1.5	0	100	.2168	.2335	.0798	
25	0.0	1.5	0	100	.2158	.2278	.0722	*

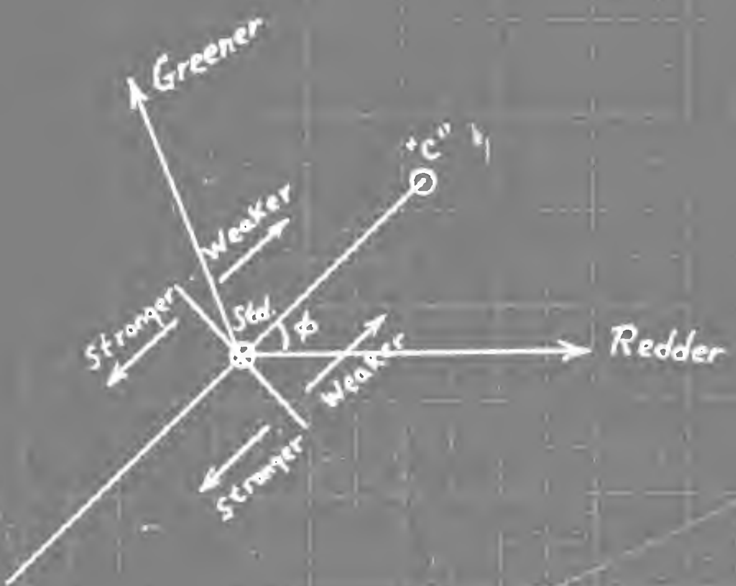
* - Point falls inside the ellipse


Blue
.2150
x
.2200
.2250
.2400
y
.2300
Each Circle = .0006 Radius

	x	y
Ill. C.	.3163	.3101
Std.	.2267	.2160
	.0896	.0941

$$\tan\phi = \frac{.0896}{.0941} = .9521$$

$$\phi = 43°35'$$



C. Judging the Samples

A method similar to that described by Davidson[2] was chosen. The samples were marked face and back and identified by a small tag on the back. The twenty-five samples were offered independently to five colorists to judge. Each colorist judged the sample three times with respect to the standard. They did not know when they were getting a sample they had judged previously. The data were tabulated as illustrated on page 48.

A chart such as the example tabulation was drawn up for each sample. With the tabulation sheets and the file of samples before him, the recorder removed the samples from their envelopes and passed them one at a time to the colorist who judged each one with respect to the standard in any manner he saw fit. The recorder tabulated each judgment by the colorist by placing a check mark in the appropriate space and returned the sample to its proper envelope. When all the samples had been judged once, the procedure was repeated a second and third time. The order in which the colorist received the samples was randomized on each repeat of the routine. Each colorist took a short rest period after judging several samples to minimize

fatigue and eye strain. A longer period of time was taken off between repeats of the routine.

The MacBeth light with approximate illuminant C (7500° K) was used in order that all judgments would be standardized in this respect. C.A.E. and C.L.E. used the light located in the Textile Chemistry and Dyeing Department of Lowell Textile Institute. G.H., J.C., and F.R. used the light located at the Derby Laboratories.

METHOD OF TABULATION OF VISUAL DATA

Example Tabulation: (one colorist - one sample)

Colorist: C.A.K. Date: 3/10/51	REPEAT	Brightness B	D	O	Hue R	Y	G	B	O	Strength W	S	O
Sample #6	1	x					x				x	
	2	x			x						x	
	3	x			x					x		

Composite Tally: (all colorists - one sample)

Sample #6	Brightness B	D	O	Hue R	Y	G	B	O	Strength W	S	O
C.A.L.	111			11		1			1	11	
C.L.K.	111			111					1	11	
G.H.	111			11				1			111
J.C.	111			111						111	
P.F.	11		1	111					111		
Total	14		1	13		1		1	5	7	3
Average	14		1	12				1		2	3

Legend:

Brightness: B - Brighter than the standard
D - Duller than the standard
O - A match to the standard

Hue: R - Redder than the standard
Y - Yellower than the standard
G - Greener than the standard
B - Bluer than the standard
O - A match to the standard

Strength: W - Weaker than the standard
S - Stronger than the standard
O - A match to the standard

COMPOSITE TABULATION OF VISUAL DATA

	Brightness			Hue					Strength		
Sample #1	B	D	O	R	Y	G	B	O	W	S	O
C.A.E.		3				3			3		
C.L.H.	2	1				3			3		
G.H.		2	1			3			3		
J.C.	1	1	1			3			3		
P.R.	2		1			3					3
Total	5	7	3			15			12		3
Average		2	3								
Sample #2											
C.A.E.	2	1				3			3		
C.L.H.	1	2				3			3		
G.H.		2	1			3			3		
J.C.	1		2			3			1		2
P.R.	2		1			3			3		
Total	6	5	4			15			15		2
Average	1		4								
Sample #3											
C.A.E.	3			3					3		
C.L.H.	3			3					2		1
G.H.	3			2				1	2		1
J.C.	3			3					1		2
P.R.	1		2	3					1	1	1
Total	13		2	14				1	9	1	5
Average									8		5
Sample #5											
C.A.E.	3			3					2	1	
C.L.H.	3			2		1			2	1	
G.H.	3			2				1	2		1
J.C.	3			3					1		2
P.R.	2		1	2			1		3		
Total	14		1	12		1	1	1	10	2	3
Average				10				1	8		3

	Brightness			Hue					Strength		
Sample #6	B	L	O	R	Y	G	B	O	W	S	O
C.A.L.	3			2		1			1	2	
C.L.H.	3			3					1	2	
G.H.	3			2				1			3
J.C.	3			3						3	
P.R.	2		1	3					2		
Total	14		1	13		1		1	5	7	3
Average				12				1		2	3
Sample #7											
C.A.L.	3			3					1	2	
C.L.H.	3			3					1	2	
G.H.	3			3							3
J.C.	3			3						2	1
P.R.	3			3					1		2
Total	15			15					3	6	6
Average										3	6
Sample #8											
C.A.L.	3					3			3		
C.L.H.	3					3			3		
G.H.	2		1			3			3		
J.C.	3					2		1	3		
P.R.	3					3			3		
Total	14		1			14		1	15		
Average											
Sample #9											
C.A.L.	3					3			3		
C.L.H.	3					3			3		
G.H.	2	1				3			3		
J.C.	1		2			2		1	1		2
P.R.	3					3			3		
Total	12	1	2			14		1	13		2
Average	11		2								

	Brightness			Hue					Strength		
Sample #10	B	D	O	R	Y	G	B	O	W	S	O
C.A.E.	3					2		1	3		
C.L.H.	2		1			2		1	3		
G.H.	3					1		2	3		
J.C.	3			2				1	3		
P.R.	2		1	1				2	3		
Total	13		2	3		5		7	15		
Average						2		7			
Sample #11											
C.A.E.	2	1		3					3		
C.L.H.	3			3					1	2	
G.H.	3			3					3		
J.C.	3			3						2	1
P.R.	3			3					1		2
Total	14	1		15					8	4	3
Average									4		3
Sample #12											
C.A.E.	3			3					3		
C.L.H.	3			3					1	2	
G.H.	1		2	3					3		
J.C.	3			3							3
P.R.	2		1	3					2		1
Total	12		3	15					9	2	4
Average									7		4
Sample #13											
C.A.E.	3			3					3		
C.L.E.	3			3					2		1
G.H.	3			3					3		
J.C.	2		1	3					1	1	1
P.R.	2		1	3					2		1
Total	13		2	15					11	1	3
Average									10		3

	Brightness			Hue					Strength		
Sample #14	B	D	O	R	Y	G	B	O	W	S	O
C.A.E.	1	2				3			3		
C.L.H.	2	1				3			3		
G.H.		1	2			3			3		
J.C.		2	1			2		1	3		
F.R.	3					1	2		3		
Total	6	6	3			12	2	1	15		
Average	0	0	3			14		1			
Sample #15											
C.A.E.	1	2				3			3		
C.L.H.	2	1				3			3		
G.H.	3					1		2	3		
J.C.	2	1				3			2		1
F.R.	3					1	2		3		
Total	11	4				11	2	2	14		1
Average	7		0			13		2			
Sample #16											
C.A.E.	2	1				3			3		
C.L.H.	3					3			3		
G.H.	1		2			3			3		
J.C.	1	1	1			2		1	3		
F.R.	3					2	1		3		
Total	10	2	3			13	1	1	15		
Average	8		3			14		1			
Sample #17											
C.A.E.	3					3			3		
C.L.H.	1	2				3			3		
G.H.			3			3			3		
J.C.		2	1			3			3		
F.R.	3					2	1		3		
Total	7	4	4			14	1		15		
Average	3		4			15					

	Brightness			Hue					Strength		
	B	D	O	R	Y	G	B	O	W	S	O
Sample #18											
C.A.E.	2	1				3			3		
C.L.H.	2	1				3			3		
G.H.	1		2			3			3		
J.C.	1		2			3			3		
P.R.	2		1			3			3		
Total	8	2	5			15			15		
Average	6		5								
Sample #19											
C.A.E.	3			3					3		
C.L.H.	3			3					3		
G.H.	3							3	3		
J.C.	1	2		3					3		
P.R.	3			2				1	3		
Total	13	2		11				4	15		
Average	11										
Sample #20											
C.A.E.	3					3			2		1
C.L.H.	2		1			3			3		
G.H.	1		2					3			3
J.C.	1		2			1		2		1	2
P.R.	1		2			1	1	1	2		1
Total	8		7			8	1	6	7	1	7
Average						9		6	6		7
Sample #21											
C.A.E.	3			1		2			1	2	
C.L.H.	3			1		2			2	1	
G.H.	2		1					3			3
J.C.	3			1				2		1	2
P.R.	2		1	3					1	1	1
Total	13		2	6		4		5	4	5	6
Average				2				5		1	6

	Brightness			Hue					Strength		
Sample #22	B	D	O	R	Y	G	B	O	W	S	O
C.A.E.	3			3						2	1
C.L.H.	3			2		1			2		1
G.H.	2		1					3	1		2
J.C.	3			1		1		1		2	1
P.R.	2		1	2			1		2		1
Total	13		2	8		2	1	4	5	4	6
Average				5				4	1		6

Sample #23											
C.A.E.	3					3			3		
C.L.H.	2	1		3					1	2	
G.H.	3			3						1	2
J.C.	3			3						3	
P.R.	2		1	3						1	2
Total	13	1	1	12		3			4	7	4
Average	12		1	9				0		3	4

Sample #24											
C.A.E.	3					3			3		
C.L.H.	3					3			3		
G.H.	3							3	2		1
J.C.	2		1			3			1	1	1
P.R.	3						3		3		
Total	14		1			9	3	3	12	1	2
Average						12		3	11		2

Sample #25											
C.A.E.	3					3			2	1	
C.L.H.	3					2		1	2	1	
G.H.	3							3	2		1
J.C.	1		2	1		1		1			3
P.R.	3			1		1	1		2		
Total	13		2	2		7	1	5	9	2	4
Average						6		5	7		4

RESULTS OF VISUAL JUDGMENTS

Sample No.	NOT AVERAGED OUT B	H	S		AVERAGED** OUT B	H	S
1	d	G	W		.	G	W
2	b	G	W		O	G	W
3	B	R	w	*	B	R	w
4	Std.				Std.		
5	B	R	w	*	B	r	w
6	B	R	s	*	B	R	.
7	B	R	.s		B	R	.
8	B	G	W		B	G	W
9	B	G	W		B	G	W
10	B	.	W		B	.	W
11	B	R	w		B	R	w
12	B	R	w		B	R	w
13	B	R	W		B	R	w
14	bd	G	W		O	G	W
15	B	G	W		B	G	W
16	b	G	W		b	G	W
17	b	G	W		.	G	W
18	b	G	W		b	G	W
19	B	R	W		B	R	W
20	b	g	w.		b	g	.
21	B	r	.	*	B	.	O
22	B	r	.	*	B	r	O
23	B	R	s		B	r	.
24	B	g	W		B	G	W
25	B	g	w	*	B	g	w

Blue

Upper case = 11 or more out of 15 as indicated

Lower case = 6-10 out of 15 as indicated

O = 11 or more out of 15 as a match

. = 6-10 out of 15 as a match

* = Point falls inside ellipse

** - Refer to the composite tally.

Note - Opposite judgments are considered as cancelling one another.

Example:

	Strength W	S	O
	1	2	
	1	2	
			3
		3	
	3		
Total	5	7	3
Ave.		2	3

7 - 5 = 2

7 - 5 = 2

Convert to a 15 basis:

2 out of 5 = 6 out of 15

3 out of 5 = 9 out of 15

This sample was then judged to be a match in strength 9 out of 15 times. Its rating is (.). See legend above.

D. Comparison of Results

COMPARISON OF VISUAL BRIGHTNESS
AND SPECTROPHOTOMETRIC LUMINANCE

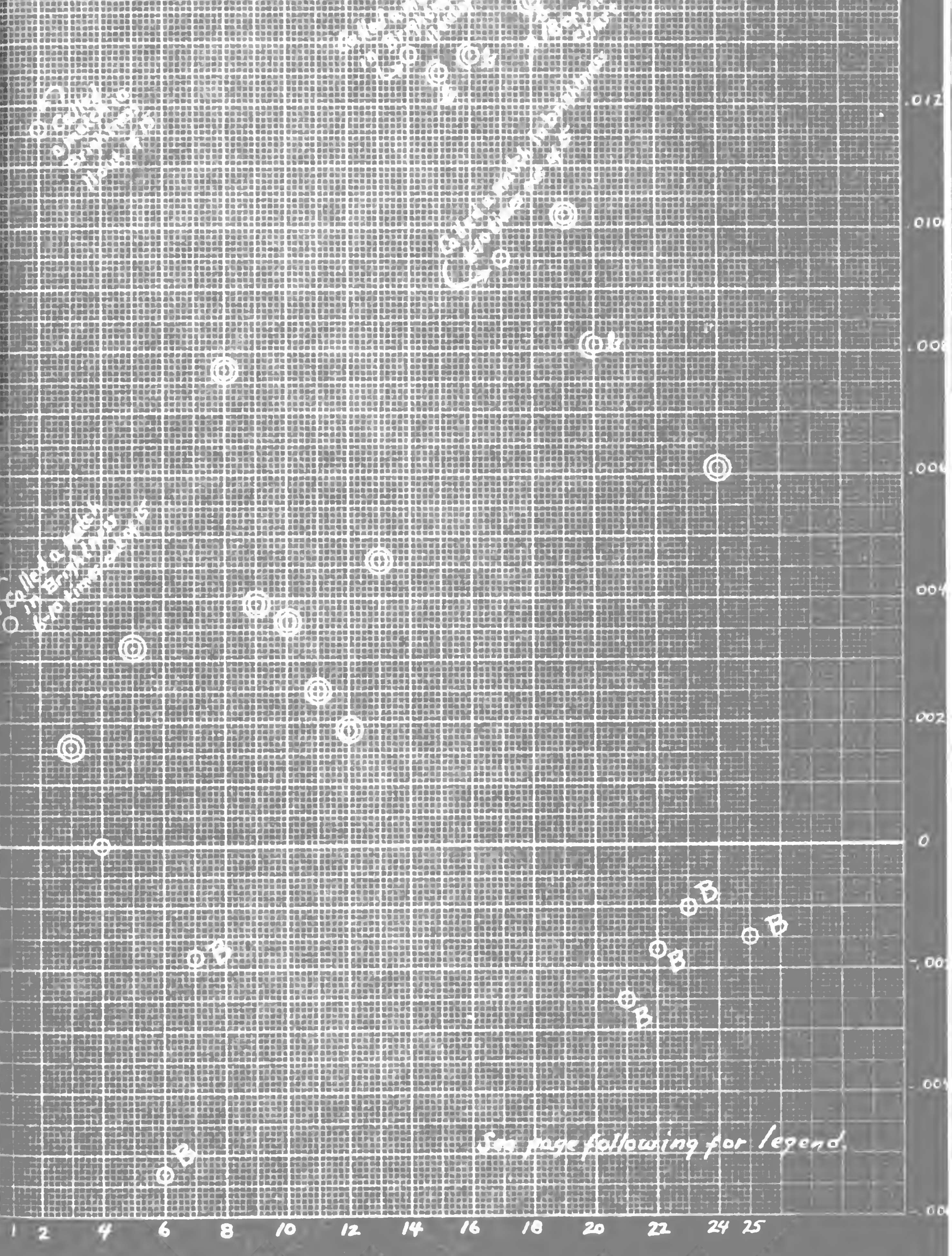
	(1)	(2)	(3)	(4)		(5)
Sample No.	Y	Y Diff. from std.	Value Munsell Renotation[14]	Colorists' Judgment Brightness		
1	.0773	+.0036	3.26	.		+0.08
2	.0853	+.0116	3.41	O		+ .23
3	.0753	+.0016	3.21	B	*	+ .03
4(Std.)	.0737	0	3.18	-		-
5	.0769	+.0032	3.25	B	*	+ .07
6	.0684	-.0053	3.06	B	*	- .12
7	.0719	-.0018	3.14	B		- .04
8	.0814	+.0077	3.34	B		+ .36
9	.0776	+.0039	3.26	B		+.08
10	.0773	+.0036	3.25	B		+.07
11	.0762	+.0025	3.23	B		+.05
12	.0756	+.0019	3.22	B		+.04
13	.0783	+.0046	3.27	B		+.09
14	.0865	+.0128	3.43	O		+.25
15	.0862	+.0125	3.43	b		+.25
16	.0865	+.0128	3.43	b		+.25
17	.0832	+.0095	3.37	.		+.39
18	.0920	+.0183	3.54	b		+.36
19	.0839	+.0102	3.39	B		+.21
20	.0818	+.0081	3.34	b		+.16
21	.0712	-.0025	3.13	B	*	- .05
22	.0720	-.0017	3.14	B	*	- .04
23	.0727	-.0010	3.16	B		- .02
24	.0798	+.0061	3.30	B		+ .12
25	.0722	-.0015	3.15	B	*	- .03

Column (3) is obtained by entering Nickerson's tables[14] with the values in Column (1) and interpolating.

Column (5) is obtained by subtracting 3.18 (value of the standard) from each value in Column (3).

Nickerson says 0.1 value step is a j.n.d.

Bellamy and Newhall[15] say 0.02 is a j.n.d. at 6 chroma.


Called a match in Brightness 6-10 times out of 15
B
See page following for legend.
0
1 2 4 6 8 10 12 14 16 18 20 22 24 25

The preceding chart, Figure IV, is a graphical representation of the brightness values as measured by the R.S.P. relative to the standard. Each numbered ordinate represents the sample of the corresponding number. The height of the ordinate represents the difference of that sample from the standard in brightness. It is a plot of the points in column (2) on page 56.

If the point is circled twice, it means that the colorists' judgment agrees with the way it plots. Those with a half circle and a lower case b were called brighter in 6-10 times out of 15. All points below the horizontal line were called brighter in 11 or more times out of 15. They are marked with a capital B. The other points are labeled.

COMPARISON OF CHROMATICITY

	Prediction From Plot			Colorists' Judgment		
1	G	W		G	W	/
2	G	W		G	W	/
3	R	S	*	R	W	½
4	-	-		-	-	-
5	R	W	*	R	W	/
6	R	S	*	R	O	½
7	R	S		R	O	½
8	G	W		G	W	/
9	G	W		G	W	/
10	R	W		O	W	½
11	R	W		R	W	/
12	R	W		R	W	/
13	R	W		R	W	/
14	G	W		G	W	/
15	G	W		G	W	/
16	G	W		G	W	/
17	G	W		G	W	/
18	G	W		G	W	/
19	G	W		R	W	½
20	G	W		G	O	½
21	G	O	*	O	O	½
22	O	O	*	R	O	½
23	R	O		R	O	/
24	G	W		G	W	/
25	G	W	*	G	W	/

/ - agrees
½ - one-half agreement
* - point falls inside the ellipse

Sixteen out of twenty-four check.
Eight out of twenty-four check one-half.
Four of the six inside the ellipse check only one-half.

CONCLUSIONS

The data obtained in this investigation indicate that this approach to the problem may lead to a satisfactory solution. A satisfactory solution depends in part upon having a large number of chromaticity points located in strategic positions. In this respect the data presented leaves something to be desired. It would have been better if more than one of the points falling inside the ellipse had been judged a match in hue and strength. See Results of Visual Judgments, page 55, and Figure II.

Without attempting an explanation to the contrary it would appear that the data presented would compel the investigator to draw the conclusion that case three (see page 20) was the fact. In which event, it would be necessary to say that the perceptibility ellipse should be smaller than the MacAdam ellipse.

However, in reality no such conclusion can be drawn without an attempt to explain matters to the contrary. It is believed that a plausible argument can be advanced to explain why some points falling inside the ellipse were judged to be not a match to the standard.

The case for argument is based upon other information derived from the same data. The information obtained independently supports the views of Davidson[13] in part.

Figure IV, page 57, bring to attention the fact that apparently there is little correlation between the brightness judgments of the colorists and the luminance as measured by the spectrophotometer. Six samples (6, 7, 21, 22, 23, and 25) were judged by the colorists in eleven or more times out of fifteen to be brighter than the standard, whereas they were found to be lower in luminance in every case by measurement on the spectrophotometer. Ten samples (3, 5, 8, 9, 10, 11, 12, 13, 19, and 24) were judged brighter in eleven out of fifteen times and measured higher in luminance. Three (15, 16, and 20) were judged brighter six to ten times out of fifteen and measured higher in luminance. But two (2 and 14) were called a match in brightness eleven times out of fifteen and were measured higher in luminance; and three (1, 17, and 18) were called a match six to ten times out of fifteen and measured higher in luminance. It is also worthy to note that the two which were called a match in eleven out of fifteen times were among the highest measured values of luminance of all the samples.

This brings to mind the assumption which was made in the beginning that there was correlation between strength, hue, and brightness, as a whole, and purity, dominant wave length, and luminance, as a whole. From the foregoing, the assumption apparently is not valid. On the other hand, there is general belief that the brightness and strength in one system and the luminance and purity in the other are interrelated. It is obvious that they are interrelated mathematically in the I.C.I. system, and this gives added strength to the belief that they are interrelated in the colorists' system. This interrelationship apparently is sufficient to cause the colorist to confuse the two, or at least to report one in terms of the other. For example, sample #25 was judged (B w), brighter and weaker, whereas the same sample was measured lower in luminance and only very slightly weaker. It is conceivable that the colorists noted that the sample was weaker and reported it in terms of brightness. Another example, sample #5 was also judged (B w), brighter and weaker. It was measured higher in luminance and even "less weaker" than #25. This is the opposite case from that above. The brightness being noted, the colorists may have reported

this in terms of weakness. A further example of this is found in the analysis of sample #3, which was judged (B w), brighter and weaker, whereas it was measured higher in luminance and definitely stronger. In this case it appears that the notice of brightness compelled the colorist to call a strong sample weak.

Again in sample #6, the colorists called it (B s.), brighter and "stronger-to-a-match". This sample was measured lower in luminance and stronger. There is no known reason why the sample was called brighter instead of duller, but this preconceived idea of brightness was manifest in calling a strong sample a match, i.e., going towards the weak direction. It is possible, however, that the reason may be that the colorists tend to judge all samples brighter than the standard subconsciously because of the standard's being handled more often than any one sample and having more chance to become soiled and dulled.

For want of a better reason, the tendency may be laid to the incompatibility between the psychological sensations with psychophysical measurements. Until this dilemma is resolved, it may offer an explanation for the fact that some of the points inside the ellipse and very

close to the standard were judged to be not a match. At this point it may be well to note that a strategic position for a point would be one in which the interrelationship between brightness and strength was inoperative. This is, of course, hypothetical.

These differences, real or imaginary, must be resolved in order to arrive at an acceptable correlation between the factors of the two systems. It is believed that this unfortunate situation will not hamper progress along these lines. For, recognizing the facts, the pitfalls of unsound conclusions or despair of success can be avoided.

There seems to be good correlation, as a whole, between the judged and measured results regarding hue or dominant wave length and strength or purity.

Data on page 59 shows the comparison between the prediction from the plot and the judgments of the colorists. It should be noted that the agreement is almost entirely confined to those points outside the ellipse. Of the points inside the ellipse, there is only one-half agreement in four of the six points. Points 3, 6, 21, and 22 show at least one point of disagreement. The prediction

from the plot was made with no reference to the ellipse. The reference was with respect to a line connecting the standard and illuminant C, as shown in Figure III, page 45. The lines divide the area into sections, weaker toward the illuminant point, stronger in the opposite direction, redder to the right of the standard, and bluer or greener above and to the left of the standard.

As this was a mere qualitative prediction as stated above, the agreement falls down in the region where quantitative readings become important. The region where quantitative readings become important is taken to be that within the confines of the ellipse. It is believed that the over-all correlation is misleading in that one might expect the correlation to stand in all regions equally. This appears not to be the case; instead, those comparisons where quantitative readings are important should be weighted more heavily, or else a complete new set of data should be collected exclusively in the area where quantitative readings are important.

To accept the correlation as a whole would obviate the fundamental concept that we seek the true color tolerance specification on the grounds of

perceptibility limits and not acceptability limits. It is in the region of quantitative readings that the perceptibility limits are involved. Qualitative comparisons may be sufficient for acceptability limits but not for perceptibility limits.

The foregoing argument is based also upon the apparent lack of validity of another basic assumption, that the data presented represents a group of samples whose brightness values are approximately equal. It is impossible to say at this time whether or not the assumption is valid or not, because the literature on the (j.n.d.), just noticeable difference, brightness step is not conclusive. Nickerson[14] believes this (j.n.d.) step to be about 0.1 Munsell value step. According to this idea, all the samples are not of approximately equal brightness. Note the data on page 56 and the column called Munsell Renotation Value. The range is from 3.06 to 3.54. The last column in this table shows the difference of each sample in Munsell Value Renotation from that of the standard. It will be noted, however, that the six points which fall inside the ellipse are within 0.1 value step of the standard, except #6, which is only -0.12 from the

standard. It, therefore, seems likely that the points of most importance are within the limits set up by Nickerson and can be said to be of equal brightness. Even though the remainder cannot be said to be of equal brightness, they are so far off in purity and dominant wave length that they are of relatively little importance, as regards the matter at hand.

According to Bellamy and Newhall[15], a (j.n.d.) in brightness is 0.02 at 6 chroma. If this is taken to be the fact, then none of the samples is "equal" in brightness to the standard, even the ones falling within the ellipse, but they are very close indeed (perhaps within experimental error), +0.03, +0.07, +0.12, -0.05, -0.04, and -0.03.

The over-all conclusion with regard to the above is that even though all the samples are not of approximately equal brightness, the ones of most importance are of "equal" brightness according to Nickerson and are very close to "equal" brightness according to Bellamy and Newhall.

Regarding the consistency and accuracy of the colorists, it is tentatively concluded that the consistency

within and among colorists is good, agreeing with Davidson[13] on this point. Further, it is tentatively concluded that the over-all accuracy of the colorists is good, agreeing with Davidson[13] on this point, but that the conclusion is misleading. For, as stated above, it seems that quantitative results become more important in the critical region (within the ellipse), and unless they are weighted more heavily in this region, the statistical analysis will be out of balance.

If such an over-all statistical analysis must be retained, it is strongly recommended that a system to weight the critical results be developed. This appears to be the only solution, for we will never be able to insert a yardstick into the eye-brain system of the human being in order to come up with some quantitative data.

A summary of the conclusions discussed above are shown below in tabular form:

1. If the data is taken as it is, then the MacAdam ellipse must be made smaller to denote a perceptibility limit.
2. Points falling inside the ellipse and not judged to be a match can be explained on the basis of the interrelationship between purity and brightness in the

colorists' system, and the incompatibility of psychological sensations and psychophysical measurements.

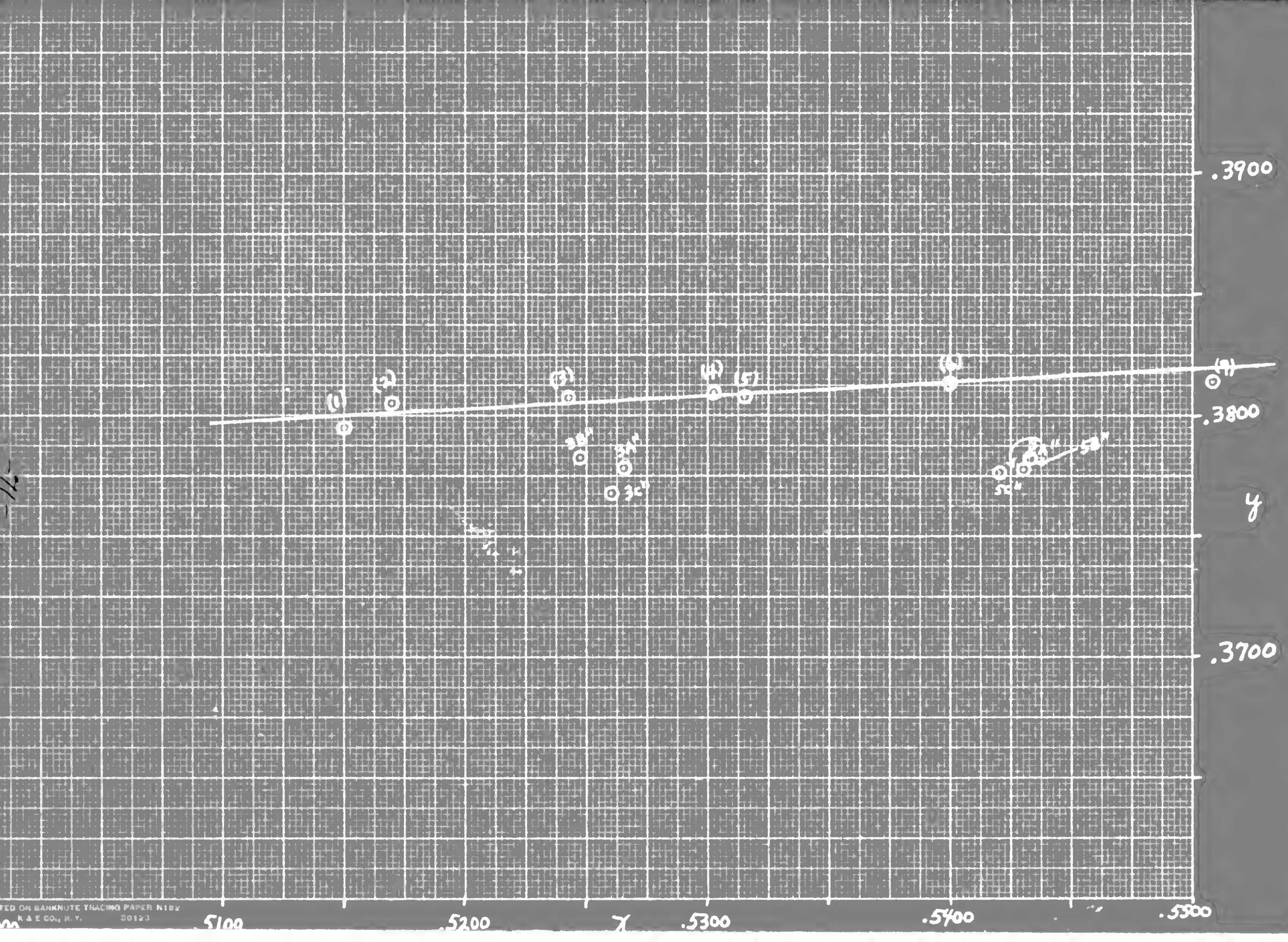
3. On the basis of (2), the MacAdam ellipse, as it is, should not be rejected as defining the perceptibility limits. But its acceptance should depend upon positive proof in the affirmative. Nothing in this investigation tends to disprove it.
4. Consistency of judgment by colorists is accepted.
5. Accuracy of judgment by colorists "as a whole" is accepted.
6. But accuracy of judgment by colorists in the region where quantitative results are important is questioned. To illustrate this point, it is conceivable that given a sample having an appreciable difference in luminance from the standard, its purity value could so be arranged that a colorist would call the sample a match in all three respects--brightness, hue, and strength (assuming no change in hue was involved).

For future work it is recommended that the general lines of this work be followed, incorporating, of course, the intelligence on the subject which seems to be coming out in the literature at an increasing rate.

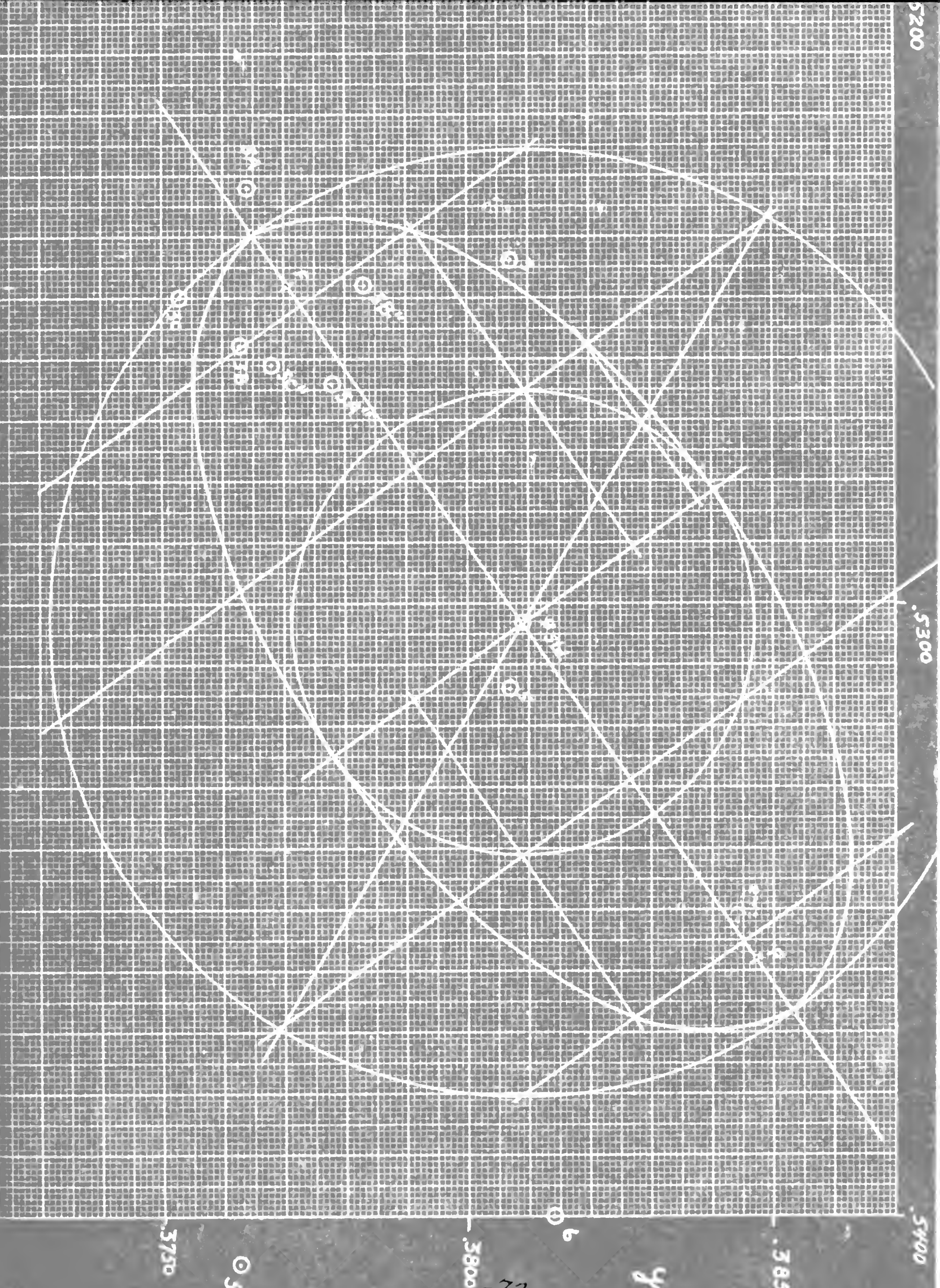
SAMPLE COMPOSITION VS. I.C.I. COORDINATES

Milling Fast Orange

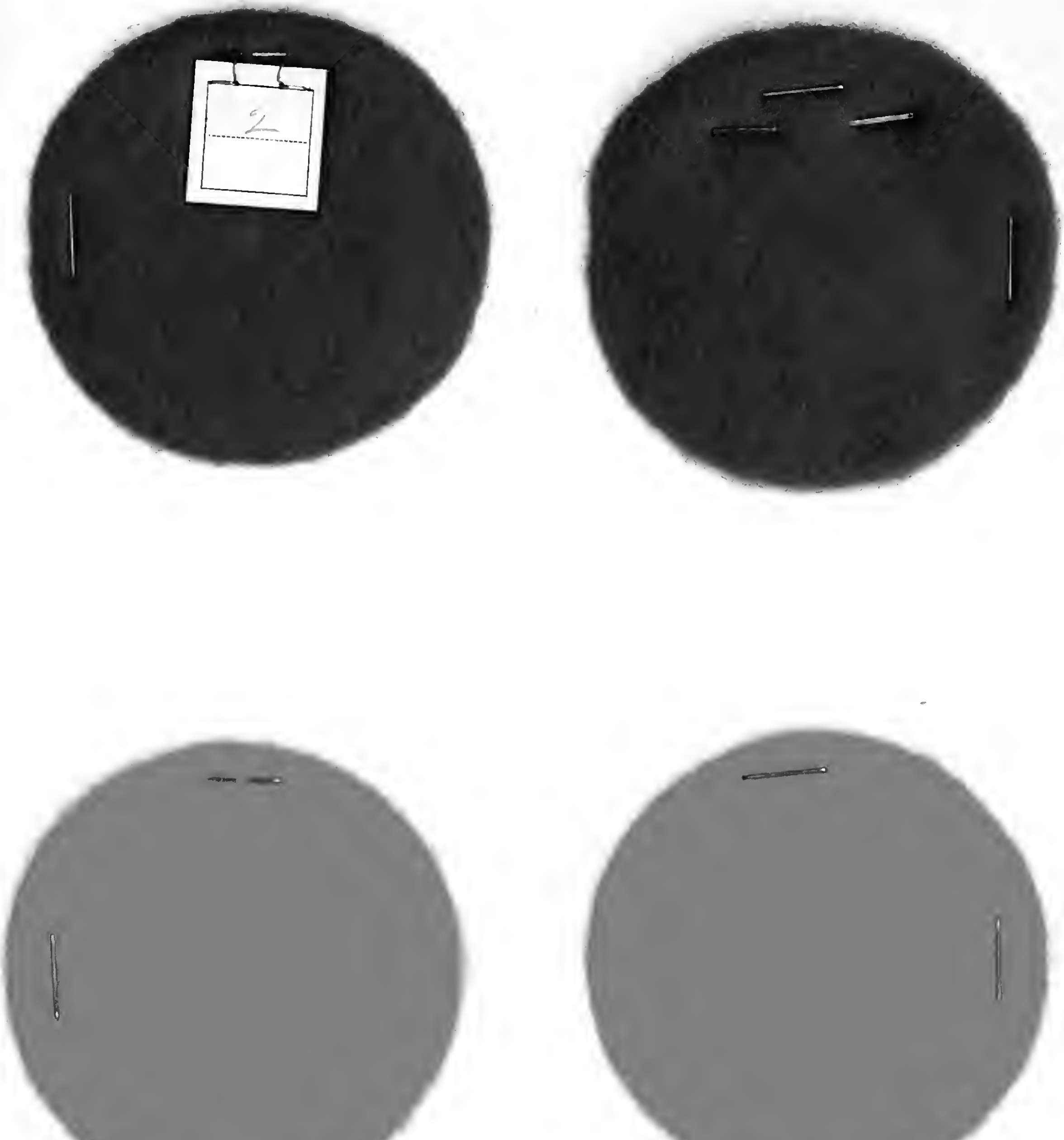
Sample No.	Wt. of Stock - gms Dye 0.75%	Wt. of Stock - gms Dye 1.0%	Blend Percent 0.75%	Blend Percent 1.0%	Chromaticity Points x	Chromaticity Points y	Brightness Y
1	1.5	0.0	100.0	0.0	.5150	.3794	.2875
2	1.2	0.3	80.0	20.0	.5169	.3806	.2840
3	0.9	0.6	60.0	40.0	.5244	.3807	.2727
3A					.5232	.3764	.2592
3A"	0.875	0.625	58.3	41.7	.5264	.3778	.2503
3B					.5258	.3763	.2594
3B"	0.85	0.65	56.6	43.4	.5248	.3783	.2515
3C					.5250	.3753	.2505
3C"	0.80	0.70	53.3	46.7	.5261	.3768	.2467
4	0.75	0.75	50.0	50.0	.5303	.3809	.2665
5	0.6	0.9	40.0	60.0	.5314	.3807	.2642
5A					.5372	.3716	.2595
5A"	0.50	1.0	33.3	66.7	.5431	.3778	.2488
5B					.5408	.3763	.2533
5B"	0.40	1.10	26.7	73.3	.5432	.3782	.2474
5C					.5426	.3757	.2476
5C"	0.35	1.15	23.3	76.7	.5420	.3776	.2462
6	0.3	1.2	20.0	80.0	.5400	.3813	.2553
7	0.0	1.5	0.0	100.0	.5508	.3814	.2408



.3900
.3800
.3700
y
.5100
.5200
.5300
.5400
.5500
x
(1)
(2)
(3)
(4)
(5)
(6)
(7)
3B"
3A"
3C"
5C"
5B"



.5200
.5300
.5400
.3750
.3800
.3850
y
5B
6


2

BIBLIOGRAPHY

(1) Handbook of Colorimetry
Hardy

(2) Visual Sensitivity to Surface Color Differences
H.R.Davidson, J. Opt. Soc. Am., Feb. 1951, pg 104

(3) Visual Sensitivity to Color Differences in Daylight
D.L.MacAdam, J. Opt. Soc. Am., May 1942, pg 247

(4) Researches in Normal and Defective Colour Vision
W.D.Wright, C.V.Mosby, St. Louis, 1947
J. Opt. Soc. Am., Sept. 1947, pg 731 (Book Review)

(5) Specification of Small Chromaticity Differences
D.L.MacAdam, J. Opt. Soc. Am., Jan. 1943, pg 20

(6) On Two Accessories of Three Dimensional Colorimetry
L.Silberstein, J. Opt. Soc. Am., Aug. 1946, pg 464

(7) The Graphical Representation of Small Color Differences
D.L.MacAdam, J. Opt. Soc. Am., Dec. 1943, pg 675

(8) Visual Sensitivity to Combined Chromaticity and Luminance Differences
Brown and MacAdam, J. Opt. Soc. Am., Oct. 1949, pg 808

(9) Construction of the General Electric Recording Spectrophotometer
Michaelson, J. Opt. Soc. Am., Oct. 1938, pg 365

(10) A Continuous, Automatic Tristimulus Integrator For Use With the Recording Spectrophotometer
Davidson and Imm, J. Opt. Soc. Am., Nov. 1949, pg 942

(11) An Investigation of Commercial Color Matching
R.L.Scott, Thesis, L.T.I., 1949

(12) Calibration and Operation of the General Electric Recording Spectrophotometer of the N.B.S.
Gibson and Keegan, J. Opt. Soc. Am., Oct. 1948, pg 372

(13) Accuracy of Visual Judgments of Color Difference on Wool Flannel
H.R.Davidson, Am. Dyestuff Rep., April 16, 1951,pg 247

(14) Tables for Computing Small Color Differences
Dorothy Nickerson, Am. Dyestuff Rep., Aug. 21, 1950, pg 541

(15) Attributive Limens in Selected Regions of the Munsell Color Solid
Bellamy and Newhall, J. Opt. Soc. Am., Aug. 1942, pg 465

On the applicability of MacAdam ellipses to small color differences in wool.

Thesis 15646
P27 Patton
On the applicability of MacAdam ellipses to small color differences in wool.